Abstract
Recently, Combined Heat and Power (CHP) systems have been utilized increasingly in power systems. With the addition penetration of CHP-based cogeneration of electricity and heat, the determination of economic dispatch of power and heat becomes a more complex and challenging issue. The optimal operation of CHP-based systems is inherently a nonlinear and non-convex optimization problem with a lot of local optimal solutions. In this paper, the Improved Shuffled Frog Leaping Algorithm (ISFLA) is used for solution of the problem. ISFLA is an improved version of shuffled frog leaping algorithm in which new solutions are produced in respect to global best solution. The ISFLA is well able to attain the optimal solutions even in the case of non-convex optimization problems. To evaluate the efficiency of the proposed method, it has been implemented on the standard test system. The obtained results have been compared with other heuristic methods. The numerical results show that the ISFLA is faster and more precise than other methods.

Keywords: Combined Heat and Power, Optimization, Economic Dispatch, Improved Shuffled Frog Leaping Algorithm

1. Introduction
In the last decades with rising fuel prices, the importance of alternative fuel discussion, increasing energy efficiency, reducing environmental pollution and increasing demand use of co-generation systems that simultaneously produce heat and power is quite very considerable. There are limitations on co-generation units is that the power generated is dependent on the production of heat. In a conventional unit, the objective of Economic Dispatch (ED) problem is to find the optimal point for the power production with minimum fuel cost such that the total demand matches the generation. However, the objective of Combined Heat and Power Economic Dispatch (CHPED) is to find the optimal point of power and heat generation with minimum fuel cost such that both heat and power demands are met while the CHP units are operated in a bounded heat versus power plane. Hence, the reference [1] shows that cogenerating plants have the ability of generating both electrical power and heat with better fuel utilization and energy efficiency. In the operation of multi-unit production systems, one of the main problems is to determine the amount of optimal production of each unit. In CHP units, the economic dispatch problem is defined as determination of the power and heat production of each unit in order to meet the needed electricity and heat with the
minimum cost, and considerable all limitations. One of the important constraints which has complicated the solution of CHPED is the dependence of electricity and heat of a unit to each other. This dependence is defined as an area which is non-convex for some of CHP units. The other factor of complexity of this debate is the non-linear, non-convex and non-differentiable function relationship between cost and power production which causes the problem to have several local optimum solutions, and the use of mathematical methods for solving this problem is limited. According to the non-convex optimization problem, none of the mathematical methods are not able to find the optimum point and ensure its optimality. Therefore in recent years, many efforts are done in to improve the solutions by using intelligent optimization methods. In [2], a method based on ant colony algorithm is presented for solving the CHPED problem. In reference [3] the improved version of the genetic algorithm is given have been proposed to solve this problem. In [4], a hybrid method based on MILP and IPM is presented for solving the CHPED problem. In [5], the harmony search algorithm is used for this solution. In this reference, the cost function is approximated by using third-degree polynomial and the effects of steam valves, which cause the non-convexity of cost function are not considered. The Particle Swarm Optimization (PSO) has been proposed in [6], [7], [8], in order to minimize the operation cost of CHP. In these papers, three types of production systems, including CHP systems, justly electricity production systems and heat production systems (boilers) are intended. In order to solve the CHPED, other evolutionary methods are implemented such as enhanced firefly algorithm [9], artificial immune system [10], [11], bee colony optimization [12], [13], differential evolution [14], hybrid time-varying acceleration coefficients-gravitational search algorithm-particle swarm optimization (hybrid TVAC-GSA-PSO) [15], time-varying acceleration coefficients particle swarm optimization (TVACPSO) [16], improved group search optimization (IGSO) [17], invasive weed optimization [18] and augmented Lagrange combined with Hopfield neural network [19]. This paper presents implementation of ISFLA as a new and efficient meta-heuristic optimization method for solving CHPED problems. Valve point effect and CHP feasible operation regions as two influential features of the CHP systems are taken into account in the presented formulation which makes the problem non-convex and hard to solve. The proposed method is implemented on a test system which the results demonstrate the superiority of the proposed method over other meta-heuristic methods.

2. Problem Formulation

The objective function of the CHPED problem is to minimize the total cost of serving the required electricity and heat demands. Commonly, the costs of the units are stated as quadratic nonlinear functions of produced electricity and heat. In addition to the complexity due to the nonlinearity of the objective function, the problem consists of considerable equality and inequality constraints. The equality constraints denote that the produced electricity and heat should be equal to the electricity and heat demands. The inequality constraints model the feasible operation boundaries of the devices. These constraints are relatively simple for power and heat-only units, i.e. limiting the energy output of these units to their min and max levels. However, for CHP units, these constraints are slightly more complicated. Since the produced electricity and heat of CHP units depend on each other, a feasible operation region is defined for each CHP...
unit. The produced electricity and heat of each CHP should be within this region. The objective function of the problem is presented in Eq. (1).

$$\text{Min } F_C = \sum_{i=1}^{np} C_i(P_i) + \sum_{j=1}^{nc} C_j(O_{j,H_j}) + \sum_{k=1}^{nk} C_k(T_k) \quad ($$/h)

(1)

Where $C_i(P_i)$, $C_j(O_{j,H_j})$ and $C_k(T_k)$ denote to fuel costs of power-only, CHP output and heat-only units respectively. The indices of power-only, CHP and heat-only units are denoted as $i$, $j$ and $k$, respectively, and $np$, $nc$ and $nk$ are the number of power-only, CHP and heat-only units, respectively.

The fuel cost of the power-only units is generally approximated by quadratic function as stated in Eq. (2).

$$C_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad ($$/h)

(2)

Where $a_i$, $b_i$ and $c_i$ are coefficients of fuel cost functions of the power-only units.

The cost function of the CHP units can be stated as Eq. (3).

$$C_j(O_{j,H_j}) = a_j O_j^3 + b_j O_j + c_j + d_j H_j^2 + e_j H_j + f_j O_{j,H_j} \quad ($$/h)

(3)

Where $a_j$, $b_j$, $c_j$, $d_j$, $e_j$ and $f_j$ are the cost coefficients, $O_j$ and $H_j$ are produced electricity and heat of the $j$th CHP unit, respectively.

The cost function of the boilers or heat-only units is presented in Eq. (4).

$$C_k(T_k) = a_k T_k^2 + b_k T_k + c_k \quad ($$/h)

(4)

Where $a_k$, $b_k$ and $c_k$ are the cost coefficients of the $k$th boiler.

The problem’s equality constraints are corresponding to the satisfaction of electricity Eq. (5) and heat Eq. (6) demands.

$$\sum_{i=1}^{np} P_i + \sum_{j=1}^{nc} O_j = P_d

(5)

$$\sum_{i=1}^{np} H_j + \sum_{k=1}^{nk} T_k = H_d

(6)

Where $P_d$ and $H_d$ are system electricity and heat demands. The inequality constraints represent the allowable operation regions of the units. The electricity generation of power-only units are restricted to the allowed min $P_i^{\min}$ and max $P_i^{\max}$ generation in Eq. (7).

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad i = 1, 2, \ldots, np

(7)

For the CHP units, the inequality constraints are as follows:

$$O_j^{\min}(H_j) \leq O_j \leq O_j^{\max}(H_j) \quad j = 1, 2, \ldots, nc

(8)
In spite of the power-only units, the electricity and heat production boundaries depend on each other as mentioned earlier. Therefore, the min and max heat and electricity production depends on the other one. For example, the min power output $O_j^{\text{min}}(H_j)$ is a function of the heat production. A simple type of the heat and power feasible operation regions are shown in Figure 1.

![Figure 1. A sample heat-power feasible regions for a CHP unit](image)

The heat production limits for the boilers are as follows:

$$T_k^{\text{min}} \leq T_k \leq T_k^{\text{max}} \quad k = 1, 2, ..., nh$$

(10)

Where $T_k^{\text{min}}$ and $T_k^{\text{max}}$ denote to min and max heat production of the boiler respectively.

3. The improved shuffled frog leaping algorithm

3.1. Introducing the shuffled frog leaping algorithm

The Shuffled Frog-Leaping Algorithm (SFLA) is an optimization method which is inspired by the group behavior of frogs to find a place that has the maximum food. It is a population-based search method which was first introduced in 2003 [20]. Each member of the population is called a frog in this algorithm and through applying two strategies of local and global search, the next generation of frogs is created and if the objective function is improved, they replace the existing frogs. This algorithm is generally consisting of the three stages as follows:

A. Production of the primary population

The primary population which includes $P$ frogs, is produced randomly in such a way that the position of each frog is within the solution periphery. The position of the $ith$ frog is shown as $X_i = (x_{1,i}, x_{2,i}, ..., x_{s,i})$ where $s$ shows the number of decision variables.

B. Classification

Frogs at this stage are arranged based on the fitness. Then, the entire population is divided into $m$ groups so that there are $n$ frogs in each group. Therefore, $p=m\times n$. 
Allocation of the frogs to each group is in such a way that the first frog is assigned to the first group, the second frog is assigned to the second group, and the \( m \)th frog is assigned to the \( m \)th group. Then, \((m+1)\)th frog is assigned to the first group and the procedure is continued until the whole \( p \) frogs are assigned to the \( m \) groups.

**C. Local search**

In local search, the position of the worst frog in each group is improved considering the position of the best solution in that group or even among the entire groups and by this, the mean fitness of the frogs is improved. For this purpose, the following steps are repeated for a determined number of times for each group:

**Step 1:** The best and the worst frog in a group are called \( X_b \) and \( X_w \), respectively based on the value of their position.

**Step 2:** The position of the worst frog in each group \( (X_w) \) is changed considering the position of the best frog in the group \( (X_b) \) as below:

\[
D = \text{rand} \times (X_b - X_w)
\]

Where \( D \) is the vector of the frog leap and \( \text{rand} \) denotes a random number within the range of \([0,1]\). Also, \( X_w^{\text{old}} \) and \( X_w^{\text{new}} \) are the old and new position of the worst frog, respectively. If at this step a better solution is achieved compared with the previous position, the new frog replaces the previous one and the procedure continues with the step 5. Otherwise, step 3 is followed.

**Step 3:** The best solution of all groups, \( X_g \), replaces \( X_b \) in the equation (11) and then through the equation (12), a new frog is obtained. If there is an improvement in the result, the new frog replaces the previous one and the procedure continues with the step 5. Otherwise, step 4 is followed.

**Step 4:** A new frog is randomly produced which replaces the worst frog in each group.

**Step 5:** Steps 1-4 are repeated a definite number of times.

**D. Termination**

The procedure of classification and local search is continued with until the termination criterion of the algorithm is met. Termination criterion of the algorithm is usually determined based on either the stability of the fitness changes of the best solution or repetition of the algorithm for a predetermined number of times.

### 3.2. The Improved Shuffled Frog-Leaping Algorithm

Although the SFLA has a high speed, it may be swamped in local optimal when solving non-linear and complicated mathematical models. Herefore, the ISLFA is introduced in this paper in which the search power is improved in comparison with the standard edition. The movement of the worst frog in each group \( (X_w) \) is modified in each repetition of the local search in ISLFA. Initially, the following leap vector is produced for the worst solution in each repetition:

\[
X_w^{\text{change}} = X_w + [F \times (X_{r1} - X_{r2})]
\]

(13)

Where \( X_{r1} \) and \( X_{r2} \) are two different frogs which are randomly select from among the frogs present in each group. \( F \) is the leap factor which determines the range of
differences between $X_{r_1}$ and $X_{r_2}$ and $X_g$ shows the best solution obtained until the present repetition.
Under this condition, the value of the $j$th parameter of the vector $X_{w}^{new}$ in the next repetition is determined using the following equation:

$$X_{wj}^{new} = \begin{cases} X_{wj}^{\text{change}} & \text{if rand}(\text{CR}_g \text{ or } j = rm) \\ X_{wj}^{\text{old}} & \text{otherwise} \end{cases}$$

(14)
Where $\text{rand}$ is a constant random number within the range of $[0, 1]$, $\text{CR}_g$ is the general intersection constant which is within the range of $[0, 1]$, and $rm$ is a random number selected from the range of the number of solution parameters and shows that at least one of the parameters of $X_{w}^{\text{new}}$ is selected from the values of $X_{w}^{\text{change}}$.

If the fitness value of the new solution is better than the previous solution, the new frog replaces the old one. Otherwise, equations (13) and (14) are repeated through replacing the best solution in each group ($X_b$) with ($X_g$) and also the local intersection constant ($\text{CR}_b$) with the general intersection constant ($\text{CR}_g$). If a better result is not achieved at this step, a random frog is created a needed replaced with the old frog within the space of the solution. As it is observed, the procedure of implementation of ISLFA is similar to that of the classic SFLA described in 3.1.A through 3.1.D, and only the movement of the worst frog for local search is modified in the proposed method to improve its efficiency. For this purpose, equations (11) and (12) in Section 3 are replaced with equations (13) and (14). The step-by-step procedure for implementation of ISLFA is shown in Figure (2).
3.3. Application of the ISLFA to solve the CHPED problem

In this section, an algorithm based on ISLFA for solving the CHPED problem is described below.

A. The structure of the frogs and assigning the initial values
During the initial assigning the values, the position of a group of frogs is randomly created. In this paper, the position of each frog is a solution for CHPED and its components show the output in units. Therefore, the position of the frog $j$ can be displayed as the following vector.
\[ X_j = \left[ P_{j,1}, P_{j,2}, \ldots, P_{j,\text{np}}, P_{j,\text{np}+1}, \ldots, P_{j,\text{np}+\text{nc}}, H_{j,\text{nc}}, H_{j,\text{nc}+1}, \ldots, H_{j,\text{nc}+\text{nk}} \right] \]

(15)

Where \( X_j \) show the power outputs of a conventional unit, power and heat outputs of co-generation units and heat alone units. In the process of the initial assigning of the values, the position of each frog is selected in a way that the limitation of equality and inequality constraints are met.

**B. Arrangement and classification**

Frogs are arranged according to the fitness of their position and then classified based on the strategy explained in 3.1.B.

**C. Local search**

Local search for each group is carried out according to the procedure described in 3.1.C and also based on the strategy explained in 3.2.

**D. Feasible operation region by CHP**

In CHP units power and heat output are inversely dependent, therefore these constraints are named as feasible operation region constraints. A penalizing method, proposed in [21] that infeasible solutions are penalized. When power and heat output of the CHP unit is outside its feasible region, a penalty factor is worked, depending on the minimum distance between the CHP unit output and the feasible region boundary. Figure 3 shows the minimum distance expressed. If \( ah + bp + c = 0 \) is the equation of the line WX then distance \((P_o, H_o)\) from the line WX calculated by Eq. (16). Then using Eq. (17) a penalty factor is calculated.

\[ L = \frac{|aH_o + bP_o + c|}{\sqrt{a^2 + b^2}} \]

(16)

\[ PF_i = Z \sum_{i=1}^{n} L_i \]

(17)

Where \( PF_i \) the penalty factor of \( ith \) solution and \( Z \) are constant value respectively. In this step, the penalty factor adds to cost function.

\[ f_c = F_c + PF \]

(18)

So, if \( f_c = F_c \), \( x \) is a feasible solution, else \( f_c = F_c + PF \), \( x \) is an infeasible solution.

![Figure 3. Graphical form of penalty calculation](image-url)
E. Condition of termination

In this section, condition of the termination of the algorithm is the maximum number of repetitions allowed, so that the search procedure is considered for a determined number of repetitions and eventually, the position of the best frog is considered as the solution for the CHPED problem.

4. Numerical Studies

To evaluate the effectiveness of the proposed method, in this section the results of the simulations on a test system are presented. The data of the test system, the simulations results and eventually the comparison of the results of the ISLFA with the other methods in the literature will be provided.

The test system includes a power-only unit, three CHP units and a boiler [5]. In Eq. (15) and Eq. (16) the cost functions of the power-only unit and boiler are presented respectively.

\[ C_1(p_1) = 0.000115p_1^3 + 0.00172p_1^2 + 7.6997p_1 + 254.8863 \]
\[ 35 \leq p_1 \leq 135 [MW] \]  
(15)

\[ C_5(T_5) = 0.038T_5^2 + 2.0109T_5 + 950 \]
\[ 0 \leq T_5 \leq 60 [MWh] \]  
(16)

Additionally, the cost function coefficients of the CHP units are provided in Table 1.

<table>
<thead>
<tr>
<th>Unit</th>
<th>a</th>
<th>b</th>
<th>C</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>Feasible region coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0435</td>
<td>36</td>
<td>1250</td>
<td>0.027</td>
<td>0.60</td>
<td>0.011</td>
<td>[44,0],[44,15.9],[40,75],[110.2,135.6],[125.8,32.4],[125.8,0]</td>
</tr>
<tr>
<td>3</td>
<td>0.1035</td>
<td>34.5</td>
<td>2650</td>
<td>0.025</td>
<td>2.203</td>
<td>0.051</td>
<td>[20,0],[10,40],[45,55],[60,0]</td>
</tr>
<tr>
<td>4</td>
<td>1565</td>
<td>20</td>
<td>0.072</td>
<td>0.02</td>
<td>2.3</td>
<td>0.04</td>
<td>[35,0],[35,20],[90,45],[90,25],[105,0]</td>
</tr>
</tbody>
</table>

The electricity and heat demands of the test system are 250 MWh and 175 MWh-th, respectively. Numerical studies were done using a software module based on the proposed algorithm in MATLAB. ISLFA parameters used for the numerical studies are shown in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>100</td>
</tr>
<tr>
<td>Group number</td>
<td>10</td>
</tr>
<tr>
<td>The number of repetitions of the local search in each group</td>
<td>10</td>
</tr>
<tr>
<td>Mutation factor</td>
<td>0.80</td>
</tr>
<tr>
<td>Global cross constant</td>
<td>0.30</td>
</tr>
<tr>
<td>Local cross constant</td>
<td>0.85</td>
</tr>
</tbody>
</table>
The results of the ISLFA and its comparison with the methods GA, HS, TVAC-PSO and GSA are provided in Table 3. According to the results, the proposed method obtains a better solution for the problem compared with the other methods. Note that, the solutions of the proposed method are supply both electricity and heat demands and in the feasible operation regions of the CHP units.

**Table 3. Comparison of simulation results for the test system**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>119.22</td>
<td>134.67</td>
<td>135</td>
<td>135</td>
<td>135</td>
</tr>
<tr>
<td>O2</td>
<td>45.12</td>
<td>52.99</td>
<td>40.01</td>
<td>39.99</td>
<td>40</td>
</tr>
<tr>
<td>O3</td>
<td>15.82</td>
<td>10.11</td>
<td>10.03</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>O4</td>
<td>69.89</td>
<td>52.23</td>
<td>64.94</td>
<td>64.98</td>
<td>65</td>
</tr>
<tr>
<td>H2</td>
<td>78.94</td>
<td>85.69</td>
<td>74.82</td>
<td>74.98</td>
<td>75</td>
</tr>
<tr>
<td>H3</td>
<td>22.63</td>
<td>39.73</td>
<td>39.84</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>H4</td>
<td>18.40</td>
<td>4.18</td>
<td>16.18</td>
<td>17.89</td>
<td>14.54</td>
</tr>
<tr>
<td>T5</td>
<td>54.99</td>
<td>45.40</td>
<td>44.14</td>
<td>42.10</td>
<td>45.43</td>
</tr>
<tr>
<td><strong>Total Cost ($)</strong></td>
<td>12327.37</td>
<td>12284.45</td>
<td>12117.38</td>
<td>12117.37</td>
<td><strong>12114.69</strong></td>
</tr>
</tbody>
</table>

Table 4 presents the influence of the population size of the proposed method on the accuracy and speed of the solution.

**Table 4. The effect of the population size on the final solution of CHPED**

<table>
<thead>
<tr>
<th>Initial Population</th>
<th>Best Solution</th>
<th>Mean Solution</th>
<th>Worst Solution</th>
<th>Standard Deviation</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>12119.54</td>
<td>12117.95</td>
<td>12173.94</td>
<td>7.73</td>
<td>1.27</td>
</tr>
<tr>
<td>50</td>
<td>12116.60</td>
<td>12117.25</td>
<td>12172.68</td>
<td>3.54</td>
<td>1.96</td>
</tr>
<tr>
<td>75</td>
<td>12114.69</td>
<td>12114.94</td>
<td>12117.72</td>
<td>0.48</td>
<td>2.18</td>
</tr>
<tr>
<td>100</td>
<td>12114.69</td>
<td>12114.86</td>
<td>12116.18</td>
<td>0.25</td>
<td>2.84</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper presented, a meta-heuristic optimization method to minimize the operation cost of the heat and power systems or the CHPED problem. In the formulation, various specifications of the devices such as feasible operation region and the capacity of them are incorporated. The proposed method has effectively provided the best solution satisfying both equality and inequality constraints. For chosen the test system, ISFLA has superiority to other methods in terms of solution accuracy and computation time. The results are compared with the results of the other optimization methods such as GA, HS, PSO and GSA, which demonstrate the dominance of the ISFLA in finding the solutions with the lowest costs and respecting all problem constraints.

**References**


