

HGELS: A New Hybrid Algorithm Based on Gravitational Force for Solving Multiple Traveling Salesman Problems

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Abstract

Multiple Traveling Salesman Problem (MTSP) is one of the most popular operation research problem and is known as combinatorial optimization problems. MTSP is an extension version of the famous and widely used problem named Traveling Salesman Problem (TSP). Because of its benefices in various domains, many researchers have tried to solve that, and many methods have proposed so far. MTSP is a NP-complete problem so deterministic algorithms aren't able to solve it in a reasonable time and in most cases heuristic methods are used. We propose a combination algorithm named at HGELS to solve MTSP in this paper. Our aim is to minimize the distance traveled by salesman and also the cost of path traveling. In fact, the proposed algorithm aims to balance the cost of path and distance. Experimental results indicate that the proposed algorithm needs more time to run than other algorithms, but in most cases has obtained better results.

Keywords: MTSP, Optimization, Genetic Algorithm, GELS Algorithm

1. Introduction

MTSP is an extension version of the famous and widely used problem named traveling salesman problem. Since MTSP is able to coordinate more than one TSP, MTSP is more suitable to model the real world situations. In fact by adding additional closed constraints to the problem, we can use the MTSP model for solving problems such as multi salesman robot [1,2], vehicle routing problem and job scheduling [3,4].

MTSP is more complicated than TSP, because cities must be assigned to each salesman at first, and then the optimum order determined for each salesman. This problem is a kind of NP problems and heuristic methods are used to solve it.

So far many methods based on heuristic algorithms have proposed for MTSP, such as: classical search maps [5], Simulated Annealing [6], Artificial Neural Networks (NNs) (Kohonen-type self-organizing maps) [7], Hopfield-type NNs [8], Genetic Algorithms

(GAs) [9], Evolutionary Programming [10], Ant Colony Optimization [11], Tabu Search [12], Fine-Tuned Learning [13], and so on.

However, the research on MTSP was limited. Bektas [14] introduced comprehensive formulas and new methods for MTSP and showed that exact algorithms in [15,16] obtain always unreasonable solutions. This way, many heuristic algorithms have proposed for MTSP so far. Ryan et al. [17] used Tabu Search for solving MTSP while Qu et al. [18] proposed competition-based neural network for solving the mentioned problem.

Genetic algorithm is using many fields including MTSP. A genetic based approach is proposed in [19]. Liaw et al. [20] proposed a combination genetic algorithm based on Tabu Search for solving MTSP. Carter et al. [21] researched on chromosome structure and genetic operations to reach a suitable solution for MTSP. Mohammadpour and Yadollahi [22] combine the genetic algorithm and gravitational force to minimize the traveled distance by salesman. Combined ant colony optimization and 2-opt based method are proposed in [23] and [24] respectively for solving MTSP.

The proposed methods so far, aim at minimizing the traveled distance and didn't mention the cost to travel path. This way, in this paper we propose a combined gravitational force to solve MTSP with the aim of minimizing traveled distance by salesman and also minimizing the cost of traveling simultaneously. In fact the proposed algorithm aims to balance the cost of path and distance.

The rest of the paper is organized as follow: problem description and GELS algorithm are presented in section 2 and section 3. The proposed algorithm is in section 4. Experimental results and conclusion are presented in section 5 and section 6 respectively.

2. Problem description

Generally, there is a set of n cities (nodes) and m salesmen in MTSP, in which several cities assigned to each salesman at first, then the optimum order for salesmen to visit cities is determined such a way that the following constrains are satisfied [13]:

- 1- Each city must be visited only by one salesman and only once
- 2- Traveled distance by each salesman must be minimized
- 3- The cost of travelling path for each salesman must be minimized
- 4- The initial city is common for all salesmen
- 5- The assigned cities to salesmen must be different (don't have common cities)

The main purpose of MTSP is to assign cities to salesmen and determine the order to visit cities such a way that travelled distance by each salesman be minimized, and finally the total distance to travel all cities by salesmen is minimized. Figure 1 shows a MTSP with 7 cities and 3 salesmen.

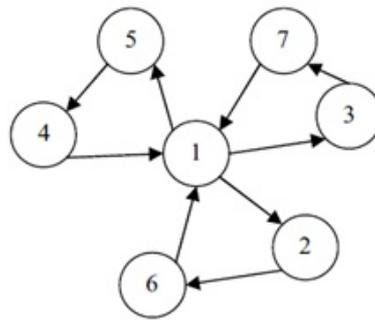


Figure 1. An example of Multiple Traveling Salesman Problem [25]

The proposed algorithm's aim is to minimize the cost of traveling path, in addition to minimize the traveled distance by salesmen. In other words, that aim is minimize distance and cost simultaneously which have considered less so far.

3. The GELS Algorithm

In 1997, GLS algorithm was proposed by Voudouris and Tesang [26] to search and solve complex problems for the first time. In 2004, Webster [27] called this method GELS (Gravitational Emulation Local Search) and was used as a powerful local search algorithm for solving complex problems. The main idea of GELS is based on gravitational force, which causes to attract objects with each other such a way that heavy object has higher gravitational force and attract low weigh objects. The attraction force between two objects depends on the distance between them.

In GELS algorithm, possible solutions in search space are divided to several categories according to their fitness's. Each of these categories named a dimension of the solution and there is initial velocity for them. Equation (1) computes the gravitational force between current solution (CU) and candidate solution (CA). This force (F) is added to velocity vector in the path of current motion. If velocity exceeds the maximum value (threshold), maximum velocity becomes the current velocity, and if the velocity becomes negative due to this force, the velocity is considered zero [28].

$$F = \frac{G(Cu - CA)}{R^2} \quad (1)$$

4. Proposed Algorithm

In this paper, a new algorithm named HGELS is proposed by combining Genetic Algorithm and Gravitational Emulation Local Search for solving Multiple Traveling Salesman Problem. The aim is to minimize the travelled distance by each salesman and the total cost of traveling path simultaneously.

The idea is combine the local search ability of GELS Algorithm and public search ability of Genetic Algorithm in order to facilitate the reaching a global optimum, and also establish a balance between traveled distance and cost of traveling path.

4.1 Chromosome Structure

In the proposed algorithm, one-dimensional array with the length of the number of cities (n) plus number of salesmen display chromosomes is used. Indexes from 0 to $n-1$ are assigned to cities and indexes from n to $n+m$ are assigned to salesmen. Figure 2 shows a sample chromosome.

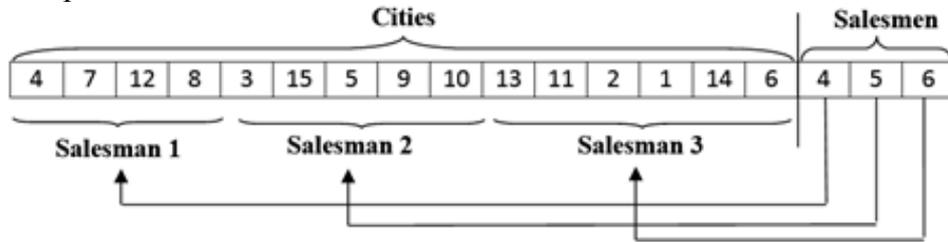


Figure 2. Chromosome structure

In chromosome structure, the number related to each salesman indicates the number of cities assigned to that salesman. For example, the number of the first salesman is 4, it means that the first salesman must travel the first fourth cities respectively.

4.2 Fitness Function

In the proposed algorithm, the aim is to minimize traveled distance and the total cost of traveling path simultaneously. Equation (2) is travelled distance by salesmen and Equation (3) is the required cost to travel path. Equation (4) is used to compute the fitness of chromosomes.

$$D = \sum_{i=1}^m \sum_{j=1}^n \sqrt{(x_{j+1} - x_j)^2 + (y_{j+1} - y_j)^2} \quad (2)$$

$$C = \sum_{i=1}^m \sum_{j=1}^n \text{cost}(c_j, c_{j+1}) \quad (3)$$

$$F(x) = \alpha D + \beta C \quad (4)$$

$$\alpha + \beta = 1 \quad (5)$$

To compute the fitness, it is necessary to set values α and β according to the importance of distance or cost. For example, if we assume that $\alpha=0$, the distance factor isn't considered in the problem and the proposed algorithm only minimizes the cost of travelling path by salesmen, and if $\beta=0$, it means that cost factor isn't considered and the proposed algorithm only minimizes the traveled distance by salesmen.

4.3 Crossover Operation

In order to keep the diversity of chromosomes, we use two kinds of crossovers.

4.3.1 Single-point crossover

In this kind of crossover, a salesman is chosen randomly and then a random point in its cities would be selected. Finally, the cities from the beginning to the random point are shifted to child chromosome and the genes from random point till the end of chromosome are converted and then shifted to the child chromosome. Figure 3 shows the single-point crossover.

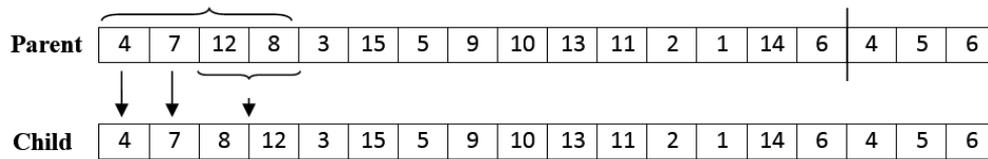


Figure 3. Single-point crossover operation

4.3.2 Two-point crossover operation

In this kind of crossover, two random points from the cities assigned to salesman are selected, and then two-point integration is performed. Figure 4 shows the two-point crossover.

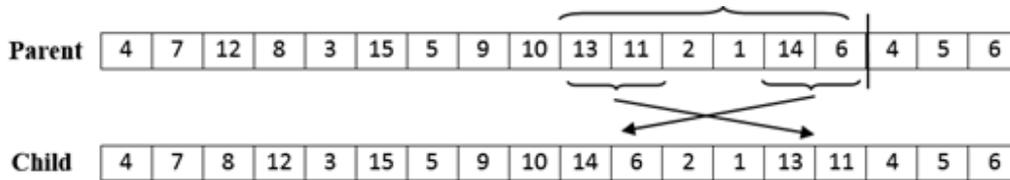


Figure 4. Two-point crossover operation

4.4 Mutation Operation

In this proposed algorithm, the exchange mutation operator is used. In this algorithm, a salesman is selected randomly and then two genes of it exchanged. Figure 5 shows the exchange mutation operator.

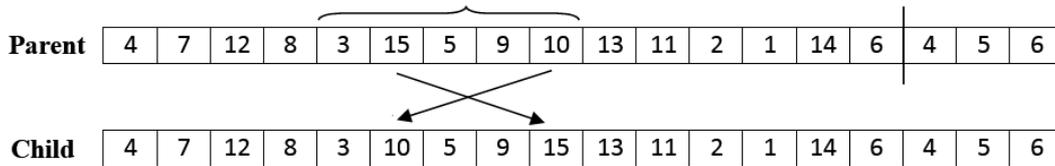


Figure 5. Exchange mutation operator

4.5 Population Optimization by Using GELS Algorithm

At this step, the proposed algorithm tries to improve the fitness of a part of chromosomes. Several chromosomes are chosen randomly and then GELS Algorithm is applied on them. In each iteration, GELS Algorithm gets each chromosome as a current solution (CU) and optimizes them by applying its operations in several steps.

Figure 6 shows the pseudo-code of proposed algorithm.

```

Procedure HGELS Algorithm
Initialize the population randomly
Evaluate each individual's fitness
Do
For  $i \leftarrow 1$  to number_of_crossover do
    Select two individual  $Parent_A$  and  $Parent_B$  from population
    Generate  $Child_x$  by crossover to  $Parent_A$  and  $Parent_B$ 
    Save  $Child_x$  to population
end_for
For  $i \leftarrow 1$  to number_of_mutation do
    Select an individual  $Parent_x$  from population
    Generate  $Child_x$  by mutation to  $Parent_x$ 
    Save  $Child_x$  to population
end_for
Perform GELS Algorithm
Evaluate each individual's fitness
While (a stop criterion is not satisfied)
return the best solution

```

Figure 6. Pseudo-code of the proposed algorithm

5. Simulation Results

To simulate algorithms, the programming language C#.Net 2014 is used. Programs were run on a computer with 2.4 GHz Pentium-IV processor and 4GB RAM.

The proposed algorithm (HGELS) is compared to GA algorithm [21]. GA algorithm optimizes only traveled distance by salesmen. So, we set parameters $\alpha=1$ and $\beta=0$ in order to the proposed algorithm minimizes only traveled distance by salesmen.

To compare the algorithms, 19 various random data test with different numbers of cities and salesmen is designed to discover small, medium and large space problems. Each data test is named as Test X_N_M. N is the number of cities, and M is the number of salesmen.

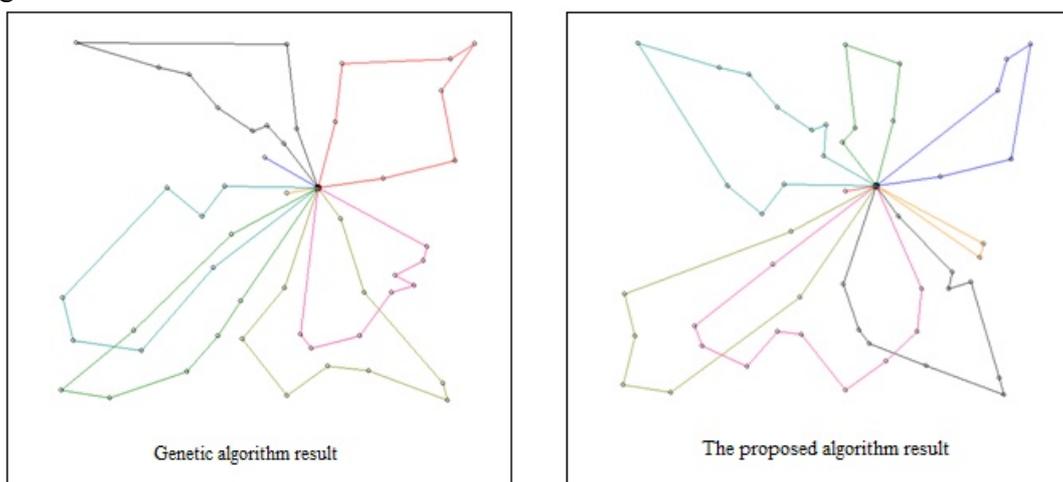
Table 1 shows the obtained results of testing the proposed algorithm (HGELS) and GA algorithm on designed data tests. In this table, the column Run Time indicates the required time to run the algorithm, and the column Best Fitness shows the best result of the algorithm.

Table 1. The results obtained from data tests

Test Number	GA [21]		HGELS	
	Best Fitness	Run Time	Best Fitness	Run Time
Test1_20_2	4636	2	4636	10
Test2_20_4	4975	2	4975	11
Test3_50_2	6582	27	6582	95
Test4_50_4	6068	32	6068	108
Test5_50_8	6857	39	6857	114
Test6_80_2	8790	118	9058	352
Test7_80_4	7941	120	8103	407
Test8_80_8	7736	136	7720	473
Test9_100_2	9397	159	9116	650
Test10_100_4	8214	178	8703	729
Test11_100_8	8447	192	8563	764
Test12_150_2	12248	370	11528	932
Test13_150_4	10534	396	10223	968
Test14_150_8	9848	412	9434	1401
Test15_150_10	10609	464	9815	1537
Test16_200_2	14386	851	13537	2167
Test17_200_4	12657	898	12049	2270
Test18_200_8	12569	1095	10521	2850
Test19_200_10	11523	1432	10862	3286

Table 1 shows that the proposed algorithm (HGELS) has obtained better results in most cases than GA algorithm. Because in the proposed algorithm, local search ability of Gravitational Emulation Local Search Algorithm is combined with public search ability of the Genetic Algorithm. The improvement is more evident in larger data tests. On the other hand, the proposed algorithm needs more time to run, because it's a combination of GELS Algorithm and Genetic Algorithm, which is a drawback of the proposed algorithm.

Figures 7-10 show the results of the proposed algorithm (HGELS) and Genetic Algorithm on various data tests.

**Figure 7. The results of Test8_80_8 data**

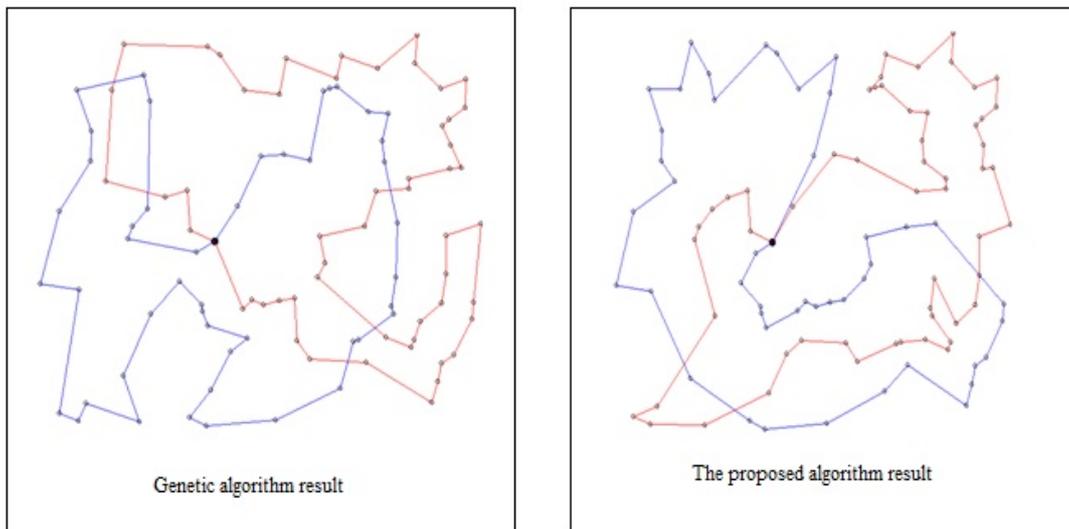


Figure 8. The results of Test9_100_2 data

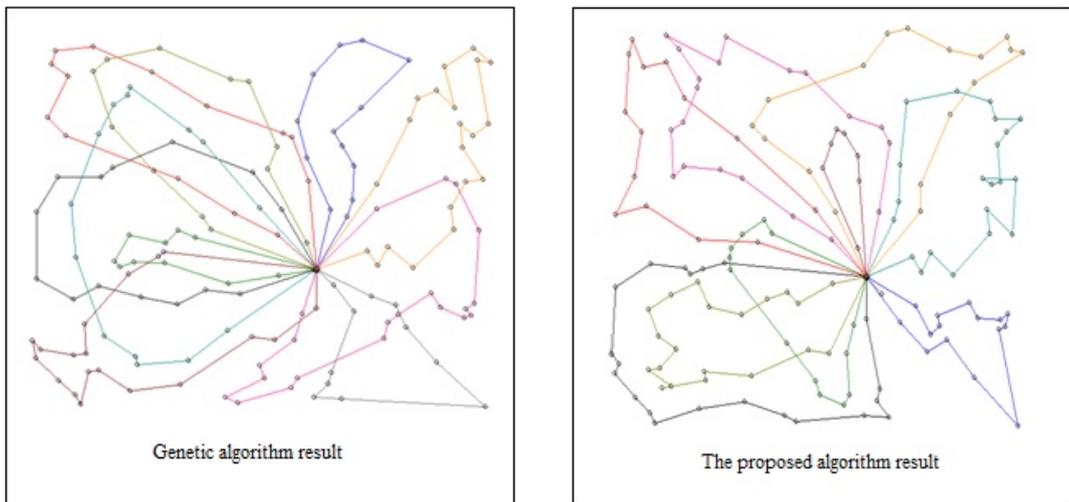


Figure 9. The results of Test15_150_10 data

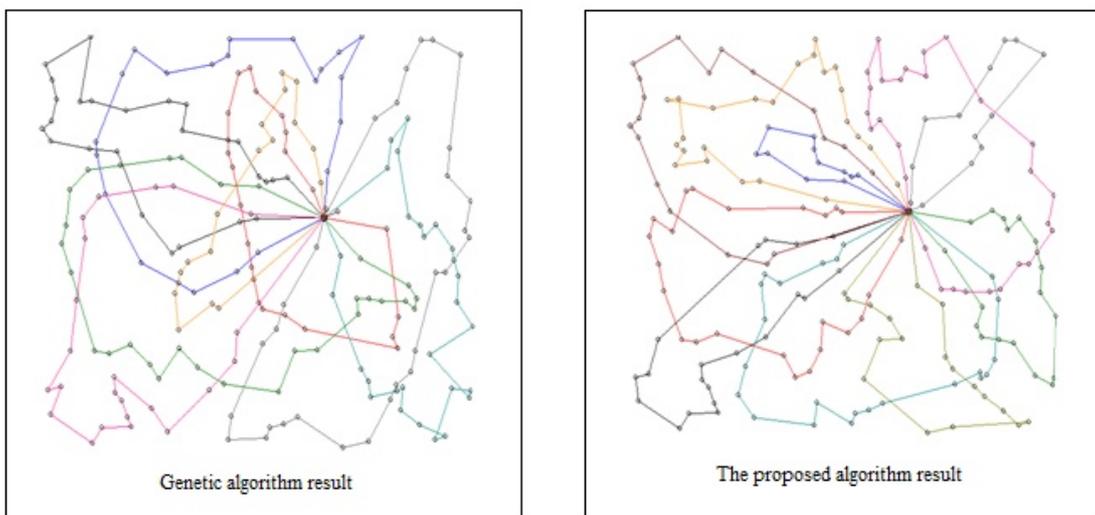


Figure 10. The results of Test19_200_10 data

Figure 11 shows the run times of the proposed algorithm (HGELS) and Genetic Algorithm. As it seen, the required time to run both algorithms is proportion to the number of cities and salesmen, and this time increases by raise them. This figure shows that the proposed algorithm needs more run time than Genetic Algorithm.

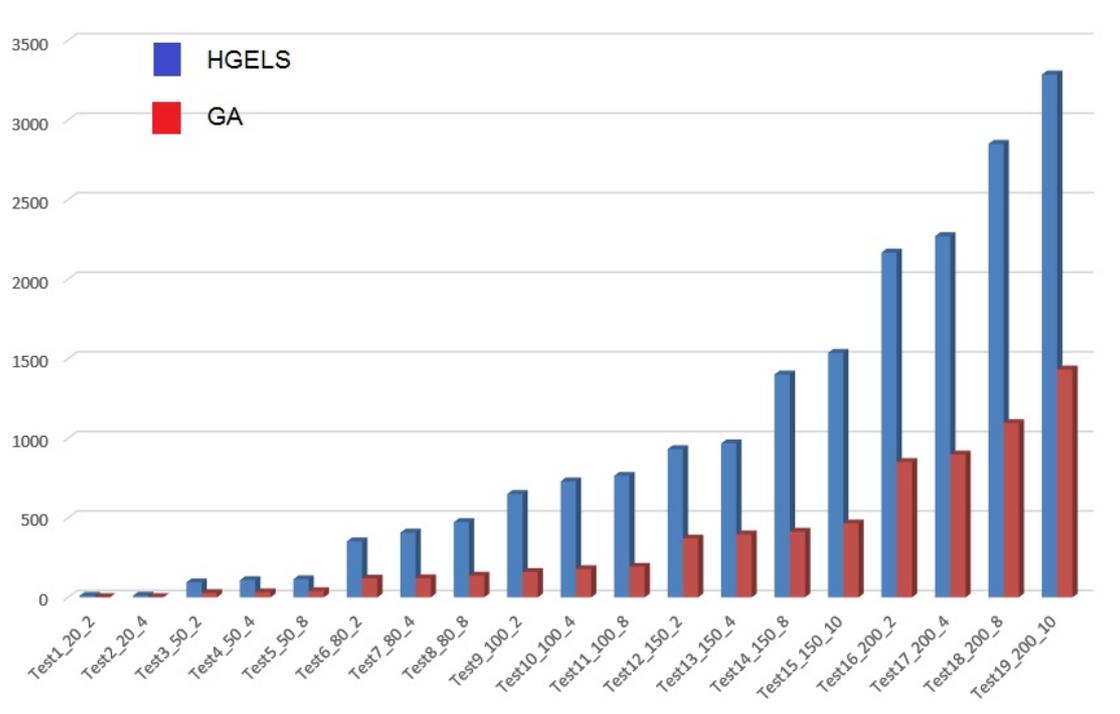


Figure 11. Required run times of the proposed algorithm and Genetic Algorithm.

Table 2 shows the results of the proposed algorithm (HGELS) in three cases $\alpha=0$, $\alpha=1$, and $\alpha=\beta=0.5$ on various data sets. This table indicates that when $\alpha=0$, the proposed algorithm minimizes the costs of traveling path regardless of traveled distance by salesmen. When $\alpha=1$, the proposed algorithm minimizes traveled distance by salesmen, regardless of the costs to travel path. Finally, when $\alpha=\beta=0.5$, the proposed algorithm balances the traveled distance by salesmen and costs to travel path, and tries to minimize both parameters.

Table 2. The obtained results from data tests for proposed algorithm

Test Number	A	B	Run Time	Distance	Cost	Fitness
Test1_20_4	0.5	0.5	14	4994	5280	5137
	1	0	11	4975	6050	4975
	0	1	11	5013	4260	4260
Test4_50_4	0.5	0.5	139	6245	80090	43167.5
	1	0	108	6068	9840	6068
	0	1	105	6371	7620	7620
Test7_80_4	0.5	0.5	473	8430	11200	9815
	1	0	407	8103	13080	8103
	0	1	416	9628	10560	10560
Test10_100_4	0.5	0.5	782	8927	10940	9933.5
	1	0	729	8703	12830	8703
	0	1	720	9280	107070	107070
Test13_150_4	0.5	0.5	1156	11059	13750	12404.5
	1	0	968	10223	15410	10223
	0	1	984	11934	13080	13080
Test17_200_4	0.5	0.5	2507	12417	16170	14293.5
	1	0	2270	12049	19630	12049
	0	1	2234	13529	15780	15780

6. Conclusion

The Multiple Travelling Salesman Problem (MTSP) is an extension of the famous and widely used problem named Traveling Salesman Problem (TSP). Since MTSP is able to coordinate more than one TSP, so MTSP is more suitable to model the real world situations. Therefore, research in this field is valuable. On the other hand, traveling salesman problem is a kind of hard problems and random search algorithms are used more than public search algorithms to solve this problem. In this paper, a new algorithm named HGELS is proposed to solve MTSP which is a combination of Gravitational Emulation Local Search Algorithm and Genetic Algorithm. The aim is to minimize the travelled distance by all salesmen and also the total cost to travel path simultaneously. The idea of this algorithm was to combine the local search ability of the GELS Algorithm and public search ability of the Genetic Algorithm to facilitate reaching to a global optimum and also balance the traveled distance and cost to travel path. Simulation results show that the proposed algorithm has obtained better results in most cases, but require more time to run. The reason of it is the combination of GELS Algorithm and Genetic Algorithm.

References

- [1] S. Sariel-Talay, T. R. Balch, and N. Erdogan, "Multiple traveling robot problem: A solution based on dynamic task selection and robust execution," *Mechatronics, IEEE/ASME Transactions on*, vol. 14, pp. 198-206, 2009.
- [2] H. Qu, S. X. Yang, A. R. Willms, and Z. Yi, "Real-time robot path planning based on a modified pulse-coupled neural network model," *Neural Networks, IEEE Transactions on*, vol. 20, pp. 1724-1739, 2009.
- [3] Sh. Shamshirband, M. Shojafar, A. R. Hosseinabadi, A. Abraham, "OVRP_ICA: An Imperialist-based Optimization Algorithm for the Open Vehicle Routing Problem", *International Conference on Hybrid Artificial Intelligence Systems (HAIS), Springer LNCS*, Vol. 9121, 221-233, 2015.

- [4] Sh. Shamsirband, M. Shojafar, A. R. Hosseinabadi, A. Abraham, "A Solution for Multi-objective Commodity Vehicle Routing Problem by NSGA-II", International Conference on Hybrid Intelligent Systems (HIS), 12-17, 2014.
- [5] J. Gu and X. Huang, "Efficient local search with search space smoothing: A case study of the traveling salesman problem (TSP)," Systems, Man and Cybernetics, IEEE Transactions on, vol. 24, pp. 728-735, 1994.
- [6] Y.-W. Chen, Y.-Z. Lu, and P. Chen, "Optimization with extremal dynamics for the traveling salesman problem," Physica A: Statistical Mechanics and its Applications, vol. 385, pp. 115-123, 2007.
- [7] M. Saadatmand-Tarzjan, M. Khademi, and H. Moghaddam, "A novel constructive-optimizer neural network for the traveling salesman problem," Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, vol. 37, pp. 754-770, 2007.
- [8] H. Qu, Z. Yi, and H. Tang, "Improving local minima of columnar competitive model for TSPs," Circuits and Systems I: Regular Papers, IEEE Transactions on, vol. 53, pp. 1353-1362, 2006.
- [9] K. Katayama and H. Narihisa, "An efficient hybrid genetic algorithm for the traveling salesman problem," Electronics and Communications in Japan (Part III: Fundamental Electronic Science), vol. 84, pp. 76-83, 2001.
- [10] D. B. Fogel, "Applying evolutionary programming to selected traveling salesman problems," Cybernetics and systems, vol. 24, pp. 27-36, 1993.
- [11] M. Dorigo, V. Maniezzo, and A. Colomi, "Ant system: optimization by a colony of cooperating agents," Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, vol. 26, pp. 29-41, 1996.
- [12] A. Misevičius, "Using iterated tabu search for the traveling salesman problem," Information technology and control, vol. 32, pp. 29-40, 2004.
- [13] S. P. Coy, B. L. Golden, G. C. Runger, and E. A. Wasil, "See the forest before the trees: Fine-tuned learning and its application to the traveling salesman problem," Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on, vol. 28, pp. 454-464, 1998.
- [14] T. Bektas, "The multiple traveling salesman problem: an overview of formulations and solution procedures," Omega, vol. 34, pp. 209-219, 2006.
- [15] G. Laporte and Y. Nobert, "A cutting planes algorithm for the m-salesmen problem," Journal of the Operational Research society, pp. 1017-1023, 1980.
- [16] A. Iqbal Ali and J. L. Kennington, "The asymmetric M -travelling salesmen problem: A duality based branch-and-bound algorithm," Discrete Applied Mathematics, vol. 13, pp. 259-276, 1986.
- [17] J. L. Ryan, T. G. Bailey, J. T. Moore, and W. B. Carlton, "Reactive tabu search in unmanned aerial reconnaissance simulations," in Proceedings of the 30th conference on Winter simulation, pp. 873-880, 1998.
- [18] H. Qu, Z. Yi, and H. Tang, "A columnar competitive model for solving multi-traveling salesman problem," Chaos, Solitons & Fractals, vol. 31, pp. 1009-1019, 2007.
- [19] A. Király and J. Abonyi, "Optimization of multiple traveling salesmen problem by a novel representation based genetic algorithm," in Intelligent Computational Optimization in Engineering, ed: Springer, pp. 241-269, 2011.
- [20] C.-F. Liaw, "A hybrid genetic algorithm for the open shop scheduling problem," European Journal of Operational Research, vol. 124, pp. 28-42, 2000.
- [21] A. E. Carter and C. T. Ragsdale, "A new approach to solving the multiple traveling salesperson problem using genetic algorithms," European journal of operational research, vol. 175, pp. 246-257, 2006.

- [22] T. Mohammadpour and M. Yadollahi, "Solving the Problem of Multiple Travelling Salesman Problem Using Hybrid Gravitational Algorithm", *International Journal on Communications (IJC)*, Volume 3, 2014, pp. 32-37.
- [23] W. Liu, S. Li, F. Zhao, and A. Zheng, "An ant colony optimization algorithm for the multiple traveling salesmen problem," in *Industrial Electronics and Applications, 2009. ICIEA 2009. 4th IEEE Conference*, pp. 1533-1537, 2009.
- [24] M. Hou and D. Liu, "A novel method for solving the multiple traveling salesmen problem with multiple depots," *Chinese Science Bulletin*, vol. 57, pp. 1886-1892, 2012.
- [25] A. S. Rostami, F. Mohanna, H. Keshavarz, A. R. Hosseinabadi, "Solving Multiple Traveling Salesman Problem using the Gravitational Emulation Local Search Algorithm", *Applied Mathematics & Information Sciences*, No. 2, 1-11, 2015.
- [26] C. Voudouris, E. Tesang, "Guided Local Search", *European Journal of Operational Research*, pp. 1-18, 1995.
- [27] B. Webster, "Solving Combinatorial Optimization Problems Using a New Algorithm Based on Gravitational Attraction", Melbourne, 2004.
- [28] A. R. Hosseinabadi, H. Siar, S. Shamshirband, M. Shojafar, M. H. Nizam Md. Nasir, "Using the gravitational emulation local search algorithm to solve the multi-objective flexible dynamic job shop scheduling problem in Small and Medium Enterprises", *Annals of Operations Research*, Vol. 229, Iss. 1, pp. 451-474, Springer 2015.