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Chaotic Time Series Prediction by Auto Fuzzy Regression Model

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Abstract

Since the pioneering work of Zadeh, fuzzy set theory has been applied to a myriad of areas. Song and Chissom introduced the concept of fuzzy time series and applied some methods to the enrolments of the University of Alabama. Thereafter we apply fuzzy techniques for system identification and apply statistical techniques to modelling system. An automatic methodology framework that combines fuzzy techniques and statistical techniques for nonparametric residual variance estimation is proposed. The methodology framework is creating regression model by using fuzzy techniques. Identification is performed through learning from examples method introduced by Wang and Mendel algorithm. Delta test residual noise estimation is used in order to select the best subset of inputs as well as the number of linguistic labels for the inputs. An experimental result for chaotic time series prediction are compared with statistical model and shows the advantages of the proposed methodology in terms of approximation accuracy, generalization capability and linguistic interpretability.

Keywords: *fuzzy techniques, statistical techniques, chaotic time series, prediction, regression.*

1. Introduction

The initial work of Zadeh concerning fuzzy set theory has been applied to a several diverse areas. Song and Chissom [1] introduced a theory for fuzzy time series and applied fuzzy time series methods [2,3] that modelled the enrolments of the University of Alabama.

In applied nonlinear time series analysis, the estimation of a nonlinear black-box model in order to produce accurate forecasts starting from a set of observations is common practice. Usually a time series model is estimated based on available data up to time t, and its final assessment is based on the simulation performance from t+1 onwards. Due to the nature of time series generated by chaotic systems, where the series not only shows nonlinear behavior but also drastic regime changes due to local instability of attractors, this is a very challenging task. For this reason, chaotic time series have been used as benchmark in several time series competitions [4,5].

Since has been suggested by Box-Jenkins [6] that the time-series ARIMA model has applied several applications in forecasting social, economic, engineering and stock problems.

For more than half-century, the Box–Jenkins methodology using autoregressive moving average model (ARMA) linear models have dominated many areas of time series forecasting [7].
Regression analysis is one of the basic tools of scientific investigation, enabling identification of functional relationship between independent and dependent variables [4]. In the classical regression analysis both the independent and dependent variables are given as real numbers. It assumes that the future values of a time series have a clear and definite functional relationship with current, past values and white noise. This model has the advantage of accurate forecasting in a short time period [8].
These techniques have limited capabilities for modelling time series data, and more advanced nonlinear methods including neural networks have been frequently applied. Fuzzy inference systems, despite its good performance in terms of accuracy and interpretability, have showed little application in the field of time series prediction as compared to other nonlinear modeling techniques such as neural networks and support vector machines.
In this paper, we propose an adaptive combining technique to predict the chaotic time series. This method combined fuzzy concept and statistical regression. The methodology proposed here is intended to apply to crisp time series.
This paper is organized as follows. In section 2, we propose a methodology framework. Section 3, illustrates the methodology through a case study for prediction chaotic time series. Finally, the conclusion is presented in Section 4.

2. Proposed model

Suppose that discrete time series as a vector $\bar{x} = x_1, x_2, ..., x_t$ that represents an ordered set of values, where "t" is the number of values in the series. The problem of predicting one future value, "x_{t+1}" using a statistical model with no exogenous inputs can be stated as follows:

$$\hat{x}_{t+1} = f_r(x_t, x_{t-1}, ..., x_{t-M+1}) \tag{1}$$

Where "$\hat{x}_{t+1}$" is the prediction of model, "f_r" and "M" are the numbers of inputs to the regressors [9].
Predicting the first unknown value requires building a model,"f_r" that maps regressor inputs (known values) into regressor outputs (predictions). When a prediction horizon higher than 1 is considered, the unknown values can be predicted following two main strategies: recursive and direct prediction.
Direct prediction requires that the process of building a "f_r" model be applied for each unknown future value. Thus, for a maximum prediction horizon H, H direct models are built, one for each prediction horizon "h" [10]:

$$\hat{x}_{t+h} = f_h(x_t, x_{t-1}, ..., x_{t-M+1}) with 1 \leq h \leq H \tag{2}$$

Direct prediction does not suffer from accumulation of prediction errors.

In order to build each model, a fuzzy inference system is defined as a mapping between a vector of crisp inputs, and a crisp output. Generally, any combination of membership functions, operators and inference model can be employed, but the selection has a significant impact on practical results. As a concrete implementation, we use the minimum for conjunctions and implications, Gaussian membership functions for inputs, singleton outputs and fuzzy mean as defuzzification method following the Mamdani defuzzification model. In this particular case a fuzzy model with M inputs for prediction horizon h is formulated as:

$$F_h(\bar{x}) = \frac{\sum_{l=1}^{N_h} \min(m_{R_l^h}, \min m_{L_l^{i,h}}(x_v))}{\sum_{l=1}^{N_k} \min m_{L_l^{i,h}}(x_v)} \tag{3}$$

Where " N_h " is the number of rules in the rule-base for horizon "h", " $m_{L_l^{i,h}}$ " are Gaussian membership functions for the input linguistic labels and " $m_{R_l^h}$ " are singleton membership functions.

The problem of building a model can be precisely stated as defining a proper number and configuration of membership functions and building a fuzzy rule-base from a data set of "t" sample data from a time series such that the fuzzy systems " $F_h(\bar{x})$ " closely predict the "h−th" next values of the chaotic time series.

We propose a methodology framework in which a fuzzy inference system is defined for each prediction horizon whit three stages. These stages are detailed in the following subsections.

A. Variable Selection

The first step in this method is choosing the optimal subset of inputs from the initial set of M inputs, with the maximum model size of M.

Delta Test is a nonparametric noise estimation method for estimating the lowest mean square error (MSE) that can be achieved by a model without over fitting the training set [9].

We use the result of the Delta Test applied to a particular variable selection as a measure of the suitability of the selection. The input selection that minimizes the Delta Test estimate is chosen for the next stages.

B. System Identification and Tuning

This stage comprises three substages that are performed iteratively and in a coordinated manner.

i. System* identification*: In this substage, the structure of the inference system (linguistic labels and rule base) is defined. For the concrete implementation analyzed in this paper, identification is performed using the W&M algorithm[10] driven by the Delta Test estimate.

This iterative identification process for increasing grid partitions of the universe of discourse stops when a system is built such that the training error is lower than the Delta Test estimate or a threshold based on the Delta Test estimate. The selection is made by comparing the error after the next stage.

ii. System Tuning: We consider an additional tuning step in the methodology as a substage separated from the identification substage. As concrete implementation for this paper we apply the supervised learning algorithm driven by the normalized MSE (NMSE).

iii.Complexity Selection: As last step, the complexity of the fuzzy model is selected depending on the DT estimate. The first (simplest) system that falls within the error range, defined by the Delta Test, is selected.

3. Case study (predicting Chaotic time series)

For the purposes of validating and illustrating the proposed methodology framework and concrete algorithms and criteria, we analyse the chaotic time series.
A short characterization of some example of chaotic time series to evaluate the prediction capability of proposed algorithms, the best data are the chaotic time series [11], generated by some linear dynamical systems. The degree of irregularity is different, from on type of series to another, depending on the sort of iterated difference equation, chaotic map or flows. In this paper, logistic time series was been tested. The logistic time series (Formula 4) generate a chaotic map with extremely short memory length. It is a difficult test for prediction algorithm. They do not exhibit cycles as we see sometimes in system practice.

$$Y(t+1)=a*y(t)*(1-y(t)) \tag{4}$$

This type of chaotic time series is a relatively easy mission for prediction algorithms. The original time series (figure 1) is split into two subsets: a training set (first 180 samples) and a test set (last 20 samples) that will be used for validation. A maximum model size of 8 parameters and a prediction horizon of 10 are considered.

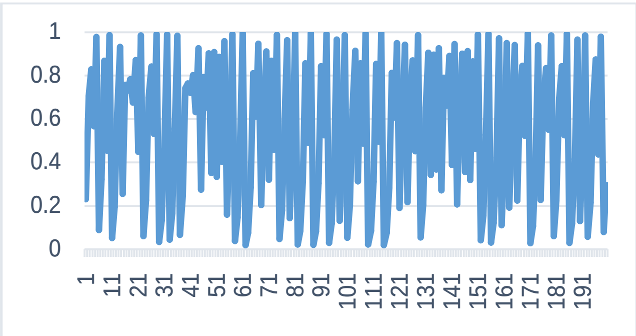

Figure.1 Number of selected variables

As first stage within our methodology, Delta Test is performed on the training set for all the possible variable selections and the one with lowest Delta Test estimate is chosen. This process is performed independently for each prediction horizon. The number of selected variables is shown in figure 2.

Figure.2 Number of selected variables

Second stage, is applying on the training set in order to identify fuzzy inference systems. These models are then tuned through supervised learning, over the training set. As last step, the first (simplest) system that falls within the error range, defined by the Delta Test, is selected.
Figure 3 shows the training and validation errors of the fuzzy regressor model. Training and test errors of statistical regressor models are also shown. Statistical regressor models were built with the same fuzzy model size.

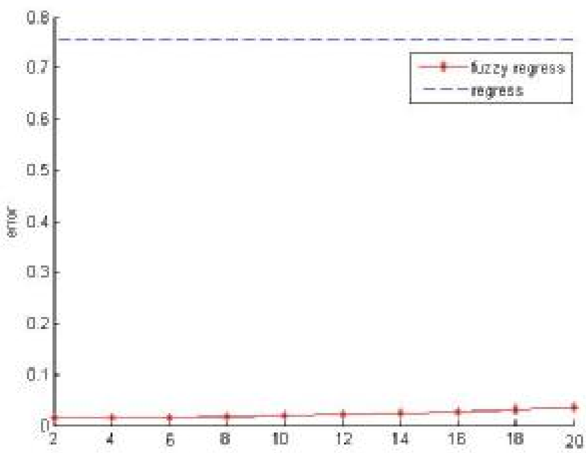

Figure.3 Comparison of our method against statistical regressor. Generalization errors of regress models (- -). Generalization errors of fuzzy models (-*-).

Figure 4 shows the predictions for the first 20 values after the training set together with a fragment of the time series.

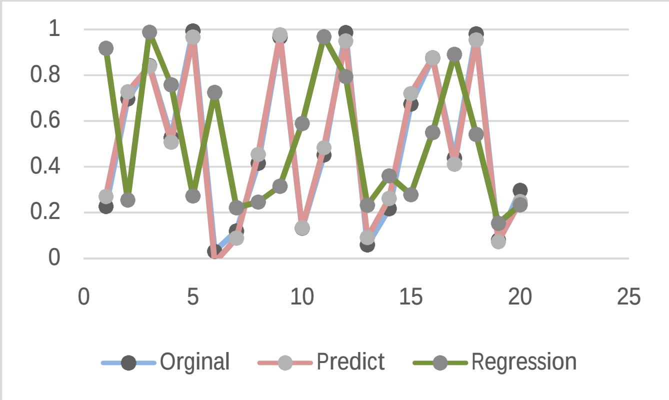

Fig.4 Prediction of 20 values after the training set by regress model and fuzzy model

4. CONCLUSION

We have developed an automatic methodology framework for long-term prediction by means of fuzzy inference systems. Experimental results for a concrete implementation of the methodology confirm good approximation accuracy and generalization capability. Linguistic interpretability for both short-term and long-term prediction as well as low computational cost has two remarkable advantages over common time series prediction methods. Also, the proposed methodology has been shown to outperform based statistical regressor model predictions in terms of approximation accuracy.

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