Optimal Type-2 Fuzzy Controller for Anti-lock Braking Systems

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Received: 2013/06/25; Accepted: 2013/09/18

Abstract

Anti-lock Braking System (ABS) is a nonlinear and time varying system including uncertainty, so it cannot be controlled by classic methods. Intelligent methods such as fuzzy controller are used in this area extensively; however traditional fuzzy controller using simple type-1 fuzzy sets may not be robust enough to overcome uncertainties. For this reason an interval type-2 fuzzy controller is developed to improve the performance of ABS in presence of uncertainty such as changing road condition. The output membership functions have been optimized by Discrete Action Reinforcement Learning Automata (DARLA) technique. Simulation results show the effectiveness of the proposed controller in comparison to type-1 fuzzy controller.

Keywords: Anti-lock Braking System (ABS), Discrete Action Reinforcement Learning Automata (DARLA), Type-2 fuzzy controller

1. Introduction

Antilock Braking System (ABS) is one of the most important safety systems in a vehicle. It prevents the wheel lock-up under critical braking condition. To achieve this goal, an anti lock braking system needs to maximize the friction force between tires and road surface [1]. During accelerating or braking, the generated friction force is proportional to the normal load of the vehicle. The ratio of this proportion is called road adhesion coefficient, and it is denoted by $\mu$. Most of the ABSs are expected to keep the vehicle slip in a desired range, where the corresponding friction force reaches its maximum value [2]. From zanten in [3], optimal performance can be achieved if the slip is kept between 8% and 30% [4], however in most ABS control strategies, optimal slip is considered as a constant value equal to 0.2 [5-6]. Researchers have improved the performance of ABS by using various algorithms such as sliding mode control [7], Adaptive control [8-9], robust control [8, 10] and fuzzy control [11-14]. Among these methods fuzzy controllers are widely used in recent researches.

Generally Type-1 fuzzy sets are used for the membership functions of ABS fuzzy controllers, however in a real anti lock braking system, uncertainty associated with the available information always occurs (for example by changing road conditions). A type-2 fuzzy controller is able to solve this problem; these controllers are known for their
ability to compensate structured and unstructured uncertainties very much. They are particularly suitable for time-variant systems with unknown time-varying dynamics such as ABS [15]. In this paper an optimal type-2 fuzzy controller is proposed to control wheel slip of an ABS.

The output of membership function of the proposed controller has been optimized by DARLA technique.

Other parts of this paper are organized as follows: The model formulation is presented in section 2. Controller designing and some related theorems are stated in section 3. Section 4 shows the simulation results of proposed design method and at the end, section 5 concludes the paper.

2. Model formulation

The mathematical model of system is developed from a quarter car model as Figure 1. The vehicle dynamics are determined by summing total forces applied to the vehicle during a braking operation.

\[ m \ddot{\xi} = -F_x \]  
\[ F_x = \mu(\lambda) F_z \]  
\[ F_z = mg \]  

Where \( m \) is quarter mass of vehicle, \( \ddot{\xi} \) is linear acceleration of the vehicle, \( \mu(\lambda) \) is friction coefficient of tire, which is a function of wheel slip (\( \lambda \)). \( F_x \) and \( F_z \) are friction force of tire and vertical force respectively.

![Figure 1. A quarter car model](image-url)
The rotational dynamic of the wheel is modeled by the equation (4):

$$J\omega = r T_s - T_b$$

(4)

Where $\omega$ is the angular speed of wheel, $T_b$ is braking moment applied to the wheel, $r$ and $J$ are Wheel radius and wheel inertia respectively.

The wheel slip for a braking operation can be found from the following equation:

$$\lambda = \frac{v - r\omega}{v}$$

(5)

Hence, a locked wheel is illustrated by $\lambda = 0$, while the free motion of a wheel is described by $\lambda = 1$[1, 16].

3. Controller designing

The employed controller in this paper is an optimal type-2 fuzzy logic system. This controller has two input variables (error of slip ratio and integral of error) and just one output with 13 consequents. The antecedent part of fuzzy If-Then rule is composed of interval type-2 membership functions as Figure 2 and every consequent part is a first order polynomial. For this fuzzy controller, Takagi- Sugeno- Kang (TSK) interface engine is used.

3.1 Interval type-2 (IT2) fuzzy logic controller

Type-1 FLSs cannot directly handle uncertainties because they use type-1 fuzzy sets that are uncertain. Type-2 fuzzy logic systems on the other hand are very useful in circumstance in which it is difficult to determine an exact membership function for a fuzzy set, Hence they can be used to handle rule uncertainties and even measurement uncertainties [17].

A type-2 fuzzy set is characterized by a fuzzy membership function i.e. membership value for each element of this set is a fuzzy set in [0 1], unlike a type-1 fuzzy set where the membership function grade is a crisp number [15].
Figure 3 shows the schematic diagram of an interval type-2 fuzzy logic system. The outputs of the inference engine are type-2 fuzzy sets, and a type reducer is needed to convert them into a type-1 fuzzy system before the defuzzification can be carried out.

![Figure 3. An Interval Type-2 fuzzy system](image)

As shown in Figure 1, an IT2 fuzzy set is bounded from the above and below by two type-1 fuzzy systems, $\tilde{X}$ and $\bar{X}$. In practice the computation in an IT2 fuzzy system can be significantly simplified. Consider the rule base of an IT2 fuzzy system consisting of $N$ rules assuming the following form [18].

$$R^n: \text{If } x_i \text{ is } X^n_i \text{ and } \ldots \text{ and } x_I \text{ is } X^n_I \text{ Then } y \text{ is } Y^n$$

$$n = 1, 2, \ldots, N$$

(6)

Where $X^n_i$ ($i = 1, \ldots, I$) are IT2 fuzzy sets and $Y^n = [\underline{y}^n, \overline{y}^n]$ is an interval, which can be understood as the centroid of an IT2 fuzzy consequent, or the simplest TSK model. For the propose simplicity in many applications we consider $Y^n = y^n = \frac{1}{2} (\underline{y}^n + \overline{y}^n)$, i.e. each rule’s consequent is a crisp number. Assume the input vector is $X' = (X'_1, X'_2, \ldots, X'_I)$.

Typical computations in an IT2 fuzzy system involve the following steps:

1) Compute the membership of $X^n_i$ on each $\tilde{X}_i$

$$[\mu_{\tilde{X}_i}^n(x_i), \mu_{\bar{X}_i}^n(x_i)] \text{ for } i = 1, 2, \ldots, I$$

$$n = 1, 2, \ldots, N$$

(7)

2) Compute the firing interval of the $n^{th}$ rule, $F^n(X')$

$$F^n(X') = [\mu_{\tilde{X}_1}^n(x'_1) \times \mu_{\tilde{X}_2}^n(x'_2) \times \ldots \times \mu_{\tilde{X}_I}^n(x'_I) \times \mu_{\bar{X}_1}^n(x'_1) \times \mu_{\bar{X}_2}^n(x'_2) \times \ldots \times \mu_{\bar{X}_I}^n(x'_I)]$$

(8)

Note that the minimum, can be used in Equation 8, instead of the product.

3) Perform type-reduction to combine $F^n(X')$ and the corresponding rule consequents. There are many such methods. The most commonly used one is the center-of-sets type-reducer.[18,19]
\[
Y_{\cos}(X) = \bigcup_{f^n \in F^n(X')} \frac{\sum_{n=1}^{N} f^n y^n}{\sum_{n=1}^{N} f^n}
\]

(9)

It has been shown that [18, 20, 21]

\[
y_j = \min_{k \in [1, N-1]} \left( \frac{\sum_{n=1}^{k} f^n y^n + \sum_{n=k+1}^{N} \bar{f}^n \bar{y}^n}{\sum_{n=1}^{L} f^n y^n + \sum_{n=L+1}^{N} \bar{f}^n \bar{y}^n} \right) = \frac{\sum_{n=1}^{L} f^n y^n + \sum_{n=L+1}^{N} \bar{f}^n \bar{y}^n}{\sum_{n=1}^{N} f^n y^n + \sum_{n=L+1}^{N} \bar{f}^n \bar{y}^n}
\]

(10)

\[
y_r = \max_{k \in [1, N-1]} \left( \frac{\sum_{n=1}^{k} f^n \bar{y}^n + \sum_{n=k+1}^{N} \bar{f}^n \bar{y}^n}{\sum_{n=1}^{R} f^n \bar{y}^n + \sum_{n=R+1}^{N} \bar{f}^n \bar{y}^n} \right) = \frac{\sum_{n=1}^{R} f^n \bar{y}^n + \sum_{n=R+1}^{N} \bar{f}^n \bar{y}^n}{\sum_{n=1}^{N} f^n \bar{y}^n + \sum_{n=R+1}^{N} \bar{f}^n \bar{y}^n}
\]

(11)

Where the switch points L and R are determined by:

\[
y_L^L \leq y^L \leq y_{L+1}^L
\]

(12)

\[
y_R^R \leq y^R \leq y_{R+1}^R
\]

(13)

And \( \{\bar{y}^n\} \) and \( \{y^n\} \) have been sorted in the ascending order respectively. \( y_i \) and \( y_r \) can be computed using the Karnik-Mendel(KM) algorithm [19].

3.2 Discrete Action Reinforcement Learning Automata (DARLA)

Reinforcement Learning Automata (RLA) method was first presented by Howell, Frost, Gordon and Wu in 1997 [22]. This method is based on interaction with the environment and employs the probability density function to find the optimum value of the decision variables of the problem [23]. In this paper, we use Discrete Action Reinforcement Learning Automata (DARLA) for optimizing output membership functions of fuzzy controller which has been stated in previous section.

In Darla, the variation limits of controller coefficient are usually divided into limits of the same length. A Discrete Probability Distribution Function (DPDF) is assigned to each limit. These DPDFs are initially set as a uniform one. Each limit is selected stochastically according to its DPDF in order to choose values for decision variables. Shapes of DPDFs are changed proportional to the fitness that corresponds to selected values. Figure 4 shows diagram of DARLA method [24].
As stated, there are 39 fuzzy controller coefficients and each variable is supposed to vary from -60 to 60. The proposed range was divided into 75 equal limits. Number of division does not severely affect on the design performance, but it must be selected large enough [24] as Equation 14.

\[
f_i^{(0)}(n) = \begin{cases} \frac{1}{75} & n = 1, 2, \ldots, 75 \\ 0 & \text{otherwise} \end{cases} \tag{14}
\]

Where \( f_i^{(k)}(n) \) is the probability of selecting \( p \)th limit in each is the probability of selecting a limit between limits of each controller coefficient at the \( k \)th iteration. After selecting limits by cumulative probability of DPDF, center of each limit is taken to construct TSFL [24] and cost \( J \) is calculated as (15).

\[
J^k = G_1 \int_0^T e \, dt + G_2 \int_0^T \left( p - p_{opt} \right) dt
\tag{15}
\]

Where \( J^k \) is cost at the \( k \)th iteration, \( T \) is simulation time and must be large enough. \( e \) is error signal, \( p \) is the braking oil pressure and \( p_{opt} \) is its optimal value. \( G_1, G_2 \) are cost element weights and considered as:

\[
G_1 = 12, \quad G_2 = 2.2842 \times 10^{-4}
\tag{16}
\]

After calculating cost, reinforcement signal \( \beta \) will be calculated as Equation 17 [22, 23, 24].

\[
\beta^{(k)} = \min \left\{ 1, \max \left\{ 0, \frac{J_{\text{mean}} - J^{(k)}}{J_{\text{mean}} - J_{\text{min}}} \right\} \right\}
\tag{17}
\]

Where \( \beta^{(k)} \) is the \( k \)th reinforcement signal, and \( J_{\text{mean}} \) and \( J_{\text{min}} \) are average and minimum of previous costs respectively. Defining reinforcement signal as (17) has a non-increasing behavior and guarantees convergence of method [24]. After obtaining reinforcement signal, DPDFs are updated by (18).
\[ f_i^{(k+1)}(n) = \alpha_i^{(k)} f_i^{(k)}(n) + \beta_i^{(k)} Q_i^{(k)} \]

\[ i = 1, 2, \ldots, n \]  

(18)

Where \( Q_i^{(k)} \) is an exponential function centered in the selected limit and defined as:

\[ Q_i^{(k)} = r_i 2^{-\left(\frac{n-n_i}{\eta_i}\right)^2} \]

(19)

Where \( n_i \) is the selected limit and \( r_i \) is a positive constant. \( \alpha_i^{(k)} \) in (18) is a normalization factor calculated as:

\[ \alpha_i^{(k)} = \frac{1}{\sum_{n=0}^{10} f_i^{(k)}(n) + \beta_i^{(k)} Q_i^{(k)}} \]

(20)

After sufficient number of iterations, the selection probability of the optimal limit for each DPDF is maximized. For each controller coefficient any limit which has the highest selection probability at the end of last iteration, is the optimum limit for that coefficient [24].

4. Simulation

In this section, simulation have been carried out using dry characteristics in order to investigate the performance of the proposed IT2 fuzzy controller compared with the common type-1 fuzzy control. First the braking was acted on the dry road surface. The braking performance is as Figure 5 and Figure 6. Next, the braking was acted on the icy road surface. The braking performance is as Figure 7 and Figure 8. From the simulation results, it can be understood that the slip ratio tracks the desired value more smoothly and with a smaller control signal comparing with a common controller.

For demonstrating the robustness of our controller, at first, we simulated the proposed ABS controller when the road surface changes from a dry road to an icy road. Figure 9 and Figure 10 show the simulation results. It can be seen that the proposed IT2 fuzzy controller tracks optimal slip better than a common controller.

In the second simulation, we consider that the road surface changes from an icy road to a dry road. Figure 11 and Figure 12 show the effectiveness of proposed controller.
Figure 5. Slip ratio for dry road

Figure 6. Control signal for dry road
Figure 7. Slip ratio for icy road

Figure 8. Control signal for icy road
Figure 9. Slip ratio in the first simulation

Figure 10. Control signal in the first simulation
5. Conclusion

In this paper, an optimized Interval Type-2 fuzzy controller has been proposed for ABS system. It is seen that the proposed controller has a better performance in tracking optimal slip against common fuzzy controller. The most attractive characteristic of the proposed controller is its robustness in the presence of the system uncertainties in the system such as changing road condition.

References


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