A Novel Noise Reduction Method Based on Subspace Division

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Abstract

This article presents a new subspace-based technique for reducing the noise of signals in time-series. In the proposed approach, the signal is initially represented as a data matrix. Then using Singular Value Decomposition (SVD), noisy data matrix is divided into signal subspace and noise subspace. In this subspace division, each derivative of the singular values with respect to rank order is used to reduce the effect of space intersections on altering the structure of important information in the signal. On the other hand, since singular vectors are the span bases of the matrix, reducing the effect of noise from the singular vectors and using them in reproducing the matrix, enhances the information embedded in the matrix. The proposed technique utilizes the Savitzky-Golay low-pass filter for noise attenuation from the singular vectors. The enhanced matrix is finally transformed to a time-series signal. The obtained results in this research indicate that the proposed method excels the other existing time-domain approaches in noise reduction.

Keywords: *Time series, Noise reduction, Singular valve, Singular vector.*

1. Introduction

Noise reduction techniques have wide applications in signal processing [1]. There are several methods for noise reduction and they can be categorized in time, frequency and time-frequency domains [2-4]. Some of existing methods can reduce the noise with a prior assumption about the signal. In other words, these methods are suitable only for specific applications and conditions. For example, in using a Low-Pass Filter (LPF), it is assumed that the noise is placed at the high frequency regions of the noisy signal. Common low-pass filters, such as those using the convolution operator, may shift the signal in time. In addition, these filters may slightly change the shape of the signal and this may be a drawback.

The Wiener filter is another important approach that is widely used by researchers and in technical applications for noise reduction in time domain. This filter is always able to reduce the noise embedded in a signal. However, the amount of noise reduction is proportionally accompanied by signal degradation [5]. In other words, Wiener filter can be used to reduce noise in a signal if the SNR is high enough (usually higher than 4 dB) [6].

When SNR in a signal is low, using Wiener filter may just transform the noise from one form to another. This is a discouraging factor in choosing the Wiener filter for noise reduction.

The authors in [6], developed a time-frequency based approach for reducing noise from a signal's time-series. This technique is based on the singular value decomposition of a matrix associated with the time–frequency representation of the signal. Indeed, in this approach the time-frequency distribution is used as a tool for representing the signal in a matrix. Then, this approach separates noise subspace and signal subspace using singular values of data matrix as criteria for subspace division. This time-frequency based technique has a good result for reducing noise in stationary and nonstationary signals. However, there are two deficiencies in the time-frequency based approach for noise reduction. A high computational time is required for representing signal in the time-frequency domain. In addition, some time-frequency distributions may not be synthesized to the time-series. Recently, time-domain based approaches for noise reduction have received a considerable attention among researchers [1, 4]. These techniques construct a data matrix in time domain (often the Hankel matrix) which represents the noisy signal. In this paper, the data matrix is divided into signal subspace and noise subspace using the SVD-based approach introduced in [11]. Then the Savitzky-Golay low pass filter is utilized to reduce noise from the singular vectors. The noise-reduced singular vectors are then used to reconstruct the matrix and eventually this noise-reduced matrix is used to extract the time-series, representing the noise-attenuated signal. We show that the SNR value is considerably improved in this new time-series signal. Results in this paper indicate that the proposed method has a better performance in noise reduction compared to other existing time-domain based approaches.

2. The Subspace Division Based Approach

In this paper, we suppose that the clean signal has been corrupted by an additive white Gaussian noise:

$$X_n = X_s + W_n \tag{1}$$

Where X_n, X_s and W_n respectively denote noisy signal, clean signal and white Gaussian noise. For $X_n(i)$, $i = 1,...,N$ representing the noisy signal, the Hankel matrix is constructed as follows:

$$H = \begin{bmatrix} X_n(1) & X_n(2) & \dots & X_n(K) \\ X_n(2) & X_n(3) & & X_n(K+1) \\ \vdots & \vdots & & \vdots \\ X_n(L) & X_n(L+1) & \dots & X_n(N) \end{bmatrix} \tag{2}$$

The singular value decomposition of matrix H with size $P \times Q$ is of the form $H = USV^T$ where $U_{P\times r}$ and $V_{Q\times r}$ are orthogonal matrices, and S is a $r \times r$ diagonal matrix of singular values with components $s_{ij} = 0 \;\; if \;\; i \neq j$ and $s_{ii} > 0$. Furthermore, it can be shown that $s_{11} \geq s_{22} \geq \ldots \geq 0$. The columns of the orthonormal matrices U and V are called the left and right singular vectors respectively.

To enhance the information embedded in the Hankel matrix, first the data matrix is divided to signal subspace and noise subspace. Then, the singular vectors of the signal subspace matrix are filtered to reduce the effects of noise from them. Finally, the enhanced data matrix is reconstructed and the noise-reduced signal is extracted.

The subspace separation can be expressed as below:

$$H = USV^T \quad = \begin{pmatrix} U_s & U_n \end{pmatrix} \begin{bmatrix} \Sigma_s & 0 \\ 0 & \Sigma_n \end{bmatrix} \begin{pmatrix} V_s^T \\ V_n^T \end{pmatrix} \tag{3}$$

$$X_s = U_s U_s^T H = HV_s V_s^T \tag{4}$$

$$W_n = U_n U_n^T H = HV_n V_n^T \tag{5}$$

where S_s and S_n represent the clean signal subspace and noise subspace, respectively. As can be seen from equation (4), we must determine a threshold point in the S matrix where lower singular values from that point can be categorized as noise subspace and hence should be set to zero. To determine this point, let plot the singular values of S matrix for a given noisy signal, respect to their indexes (see Figure 1).

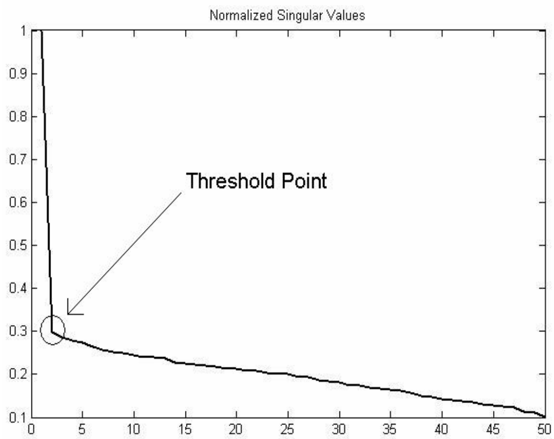

***Figure 1**. Normalized singular values of the Hankel matrix associated with a noisy signal.*

A break point can be seen clearly in Figure 1, where slop of the curve changes drastically. This threshold point can be determined by calculating derivation of the curve in each point. Our research shows that the noise subspace is mainly related to those singular values that are lower than this threshold point. Thus, we suggest setting these singular values to zero for space division.

It can be inferred from our experiments that by merely filtering the singular values, some noisy data will still be available in the signal subspace. Thus we can filter them for more noise reduction. In this study, singular vectors are treated as time-series. To reduce the effect of noise on them we use the Savitzky-Golay smoothing filter. In this approach a polynomial of degree d is fitted to N_w data points (frame size or window). Filtered singular vectors can be obtained as follows:

$$U_{ei} = F(U_{si}), \quad i = 1,\dots,P \tag{6}$$
$$V_{ei} = F(V_{si}), \quad i = 1,\dots,Q \tag{7}$$

where $F(\cdot)$ is the Savitzky-Golay smoothing filter and P ,Q indicate the size of the Hankel matrix. In the proposed noise reduction approach, the amount of noise reduction depends on the Hankel matrix size (L), the degree (d) and frame size (N_w) of the Savitzky-Golay filter. In this equation, L indicates the optimum number of rows of the Hankel matrix H.

We define a cost function to obtain a better noise reduction performance. This function depends on the above-mentioned parameters:

$$\begin{aligned} J(L\ ,d\ ,N_w) = (1-a)\left(\sum_k \left|X_e(k) - X_n(k)\right|\right) \\ + a\sum_k \left(x_e(k+1) - x_e(k)\right). \end{aligned} \tag{8}$$

At the right side of above equation, the first term indicates the Euclidian distance between the enhanced signal and the noisy signal. This distance is multiplied by a weight $(1-a)$. The second term indicates smoothness of the enhanced signal. The parameter a is a factor which determines the smoothness and must be between 0 and 1. In this paper it was experimentally set to 0.3; we will minimize this cost function using the genetic algorithm [10].

The enhanced data matrix is then obtained using:

$$H_e = U_e \Sigma_s V_e^T \tag{9}$$

where the enhanced signal X_e is extracted as follows:

$$X_e = [\,H_e(1,1)\ \ \dots\ \ H_e(1,Q)\ \ \ H_e(2,1)\ \ \dots\ \ H_e(P,Q)] \tag{10}$$

3. Simulation Results

In this section, several experiments have been carried out on multi-component periodic signals as well as linear FM (LFM) signals to show the considerable performance of the proposed approach. These synthetic signals are corrupted by additive white Gaussian noise and the results of using each method is described in following.

3.1- Multi-component signals

$$\begin{aligned} \text{Let } x &= 0.39\sin(2p ft) + 0.75\cos(2p(7f\ )t) \\ &+ 0.93\sin(2p(2f\ )t) + 0.69\cos(2p(4f\ )t) \end{aligned} \tag{11}$$

represents a clean multi-component signal where $f = 23Hz$ and the sampling frequency is $f_s = 2.5KHz$ in this experiment. The number of samples is $N = 600$.

Results of applying different approaches on this signal are shown in Figure2. As the figure shows, the convolution-based low-pass filter (LPF) can considerably reduce the noise, but with the cost of shifting and slightly changing shape of the signal. This deformation is proportional to the filter window length. Although there is no such deficiencies in using the Wiener filter, the noise attenuation level is less than the LPF, especially at lower SNRs (see Figure3).

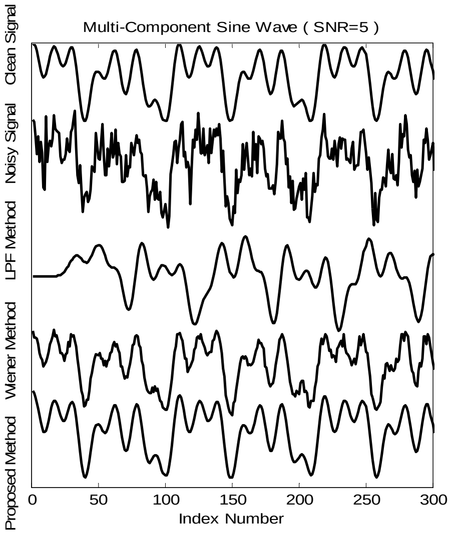

Figure 2. *Comparing performance of noise reduction techniques on multi-component signals. From top to bottom: clean signal, noisy signal, output of LPF, Wiener filter, and the proposed approach with SNR=5 dB.*

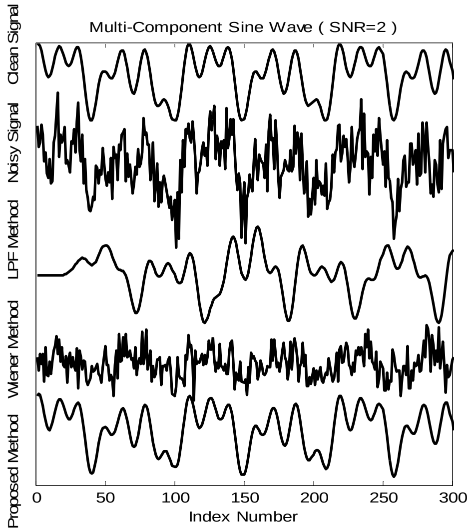

Figure 3. *Comparing performance of noise reduction techniques on multi-component signals: from top to bottom clean signal, noisy signal, output of LPF, Wiener filter, and the proposed approach with SNR=2 dB.*

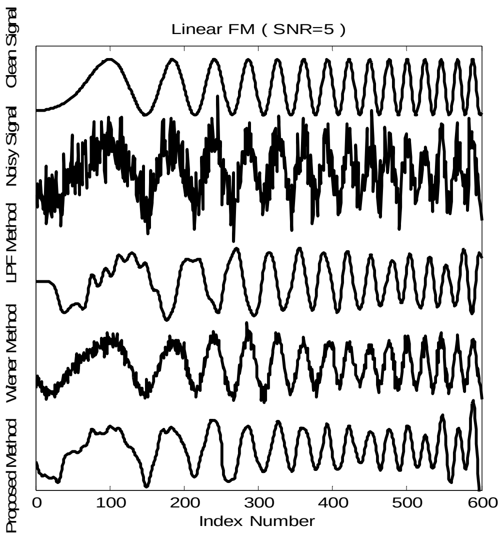

Figure 4. *Comparing performance of noise reduction techniques on LFM signals: from top to bottom clean signal, noisy signal, output of LPF, Wiener filter, and the proposed approach with SNR=5 dB.*

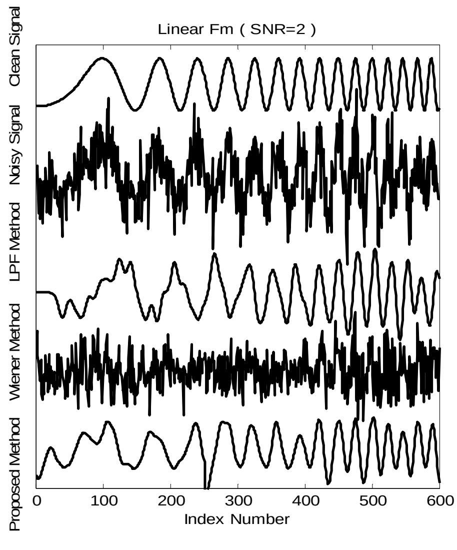

Figure 5. *Comparing performance of noise reduction techniques on LFM signals: from top to bottom clean signal, noisy signal, output of LPF, Wiener filter, and the proposed approach with SNR=2 dB.*

3.2- LFM Signals

Now we implement the different methods on linear FM signals considered as nonstationary signals. The same as previous experiment the sampling frequency is $f_s = 2.5KHz$ and the number of samples is $N = 600$. Results of this experiment are shown in Figure4. Similar to the previous experiment, the LPF shifts and slightly deforms the signal. This deformation gets worse when SNR is reduced (see Figure5). This experiment indicates that the proposed approach is suitable for a broad band of signals.

The results of Monte-Carlo simulation on 100 realizations of different SNR values for the multi-component and the LFM signals are respectively shown in Table 1 and 2, where we have compared these three methods by two famous criteria: The signal to noise ratio and the Euclidian distance. These results attest that the proposed approach has a proper performance compared to the other existing approaches in noise reduction.

Table 1. *The Monte-Carlo simulation on 100 realizations of different SNR values for the pre-mentioned multi-component signal*

Method	Initial SNR (dB)	Final SNR	Initial E. Distance	Final E. distance
Wiener	5	12.6322	288	156
	2	0.6492	396	659
	1	-4.6039	455	1004
	0	-43.755	514	8922
LPF	5	7.8511	288	686
	2	4.3301	396	818
	1	2.8735	455	855
	0	1.5648	514	898
Proposed approach	5	18.0638	288	67
	2	13.7242	396	111
	1	11.1488	455	153
	0	9.9826	514	183

Table 2. *The Monte-Carlo simulation on 100 realizations of different SNR values for the LFM signal.*

Method	Initial SNR (dB)	Final SNR	Initial Euclidean Distance	Final Euclidean distance
Wiener	5	12.8588	303	118
	2	0.2247	428	782
	1	-4.0039	490	980
	0	-14.022	566	2448
LPF	5	7.8663	303	514
	2	3.8812	428	723
	1	1.011	490	790
	0	0.1428	566	877
Proposed approach	5	16.0560	303	95
	2	14.0223	428	116
	1	11.1012	490	161
	0	7.8450	566	282

4. Conclusion

The technique proposed in this paper is a novel approach for enhancing noisy signals in time domain. In this paper the noise subspace is initially eliminated from the signal subspace using the SVD-based technique. Then the singular vectors are filtered utilizing the Savitzky-Golay smoothing filter. The dimensions of Hankel matrix representing the noisy signal, the polynomial degree and window size of the Savitzky-Golay filter are determined using genetic algorithm. Results in this paper indicate the considerable advantages of the proposed approach over the existing approaches for noise reduction in time domain.

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