

An Adaptive Segmentation Method Using Fractal Dimension and Wavelet Transform

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Abstract

In analyzing a signal, especially a non-stationary signal, it is often necessary the desired signal to be segmented into small epochs. Segmentation can be performed by splitting the signal at time instances where signal amplitude or frequency change. In this paper, the signal is initially decomposed into signals with different frequency bands using wavelet transform. Then, fractal dimension of the decomposed signal is computed and used as a feature for adaptively segmenting the signal. Any changes on the signal amplitude or frequency are reflected on the fractal dimension of the signal. The proposed method was applied on a synthetic signal and real EEG to evaluate its performance on segmenting non-stationary signals. The results indicate that the proposed approach outperforms the existing method in signal segmentation.

Keywords: *Segmentation, Non-stationary, Wavelet transform, Fractal dimension.*

1. Introduction

In many signal processing applications, such as automatic analysis system of EEG signal, the signal of interest is initially segmented into small epochs to consider the signal as piece-wise stationary [1]. This is performed as many of the existing signal processing techniques are only applicable to stationary signals. As another example, Network traffic anomaly detection plays an important part in network management and design. It is often difficult to detect the times when the faults occur in the network. It has been shown that an adaptive segmentation method may be useful to captures the failure points [2, 3].

In many signal processing methods, such as the one introduced in [4], the signal is divided into fixed-length small epochs. By this method, the segmented signal may not have criteria needed for the analysis system (piece-wise stationary). To solve segmentation problem with the fixed length epoch, it is necessary to employ a segmentation procedure that automatically recognizes the true segments boundaries. The automatic detection of the true boundary is known as adaptive segmentation. There are a number of segmentation techniques in the literature for automatic segmentation of signals. The adaptive segmentation algorithm suggested by Silin and Skrylev [5] uses two successive windows moving along the time series to extract a feature. A measuring

difference function is calculated through the difference of the feature(s) in the two successive windows. Adaptive segments boundaries are then obtained by local maxima of the measuring difference function. In this algorithm, the spectral value estimated via Fast Fourier Transform (FFT) is used as a feature for adaptive segmentation. Since calculation of the spectral value using FFT is inefficient, this method was later modified by Varri [6]. In [6], the measure difference function is in the form of a combination of frequency measure (F_{dif}) and amplitude measure (A_{dif}), as described below:

$$F_{dif} = \sum_{k=1}^l |x_k - x_{k-1}| \quad (1)$$

$$A_{dif} = \sum_{k=1}^l |x_k| \quad (2)$$

where l represents the window length and x_k is the k^{th} data point. The measure difference function is defined as follows:

$$G_m = x_A |A_{dif_{m+1}} - A_{dif_m}| + x_F |F_{dif_{m+1}} - F_{dif_m}| \quad (3)$$

x_A , x_F are coefficients for amplitude and frequency measures, m is the analyzed window number. To reduce false segment boundaries, Krajca purposed applying a threshold on the measure difference function [7]. Hence, the local maxima of function G above the threshold are considered as segment boundaries. In another method, a nonlinear energy operator (NEO) has been used for EEG segmentation [8]. This operator for a sequence x is defined as follows:

$$\Psi_d[x[n]] = x^2[n] - x[n-1]x[n+1] \quad (4)$$

The output of NEO is proportional to multiplication of instantaneous amplitude and frequency of input signal; therefore, two concepts of amplitude and frequency have been placed in single measurement. To further illustrate the technique, consider NEO applied on a tone signal:

$$Q_d(n) = y_d(A \cos(w_0 n + e)) = A^2 \sin^2 w_0 \quad (5)$$

In this method, NEO operator creates cross-term for multi-component signals and therefore segmentation will not be performed correctly. To improve segmentation of multi-component signals using NEO operator, wavelet transform can be used as a pre-processing step [9]. In this method, initially the signal is decomposed into different frequency bands using wavelet transform. Then, NEO is applied on the decomposed signals to find segments boundaries. Since frequency change in decomposed signals is less than the change in original signal, the cross-term problem is reduced.

In this paper, we initially decompose the original signal into different frequency bands using wavelet transform. Then fractal dimension (FD) of the decomposed signal is calculated. Finally, using variation of the FD, segments boundaries are determined.

2. Wavelet Transform Decomposition of Signals

Discrete wavelet transform (DWT) can decompose time-domain signal to different frequency bands by different time and frequency resolutions. Discrete time signal x can be decomposed to two different frequency bands using DWT as follows [10]:

$$y_h[k] = \sum_n x[n]g_h[2k - n] \quad (6)$$

$$y_l[k] = \sum_n x[n]g_l[2k - n] \quad (7)$$

Where $y_h[.]$ and $y_l[.]$ are the outputs of the high-pass and low-pass filters with $g_h[.]$ and $g_l[.]$ as impulse responses after sub-sampling by 2, respectively. The resulted components (i.e. $y_h[.]$ and $y_l[.]$) can be again filtered for more decomposing the signal. Original signal can be reconstructed as follows:

$$x[n] = \sum_k (y_h[k]g_h[2k - n] + y_l[k]g_l[2k - n]) \quad (8)$$

It may need to be noted that there is no absolute way to select a certain wavelet. The selection of wavelet transform depends on application. The selection of wavelet function which conforms to the original signal is very useful in wavelet applications[10].

3. Fractal Dimension of signals

Fractal Dimension is a powerful tool for transient detection of a signal. Fractal Dimension analysis has been repeatedly used in biomedical signals processing such as EEG analysis [11]. There are various algorithms to calculate FD of signal. In Esteller work, many major methods for FD calculating used in EEG analysis are compared [11]. Hiaguchi algorithm has a more precise estimation for FD of signal but is sensitive to noise. Katz algorithm in estimation of FD, compared to Hiaguchi, has a lower accuracy. Katz algorithm is a suitable method to distinguish epilepsy disease made by EEG signal. This algorithm has a lower sensitivity to noise. Petrosian method is not suitable for analysis of analog signal, because in reconstruction of dynamic range of synthetic FDs, acts weak. In addition this algorithm, like Hiaguchi, is sensitive to noise.

Selection of FD algorithm is application dependent [11]. In this paper, to calculate FD, we have used Katz algorithm. Therefore, FD of the curve is defined as follows [12]:

$$FD = \frac{\log_{10}(L)}{\log_{10}(d)} \quad (9)$$

Where L is the total curve length or sum of distances between successive points, d is diameter estimation of the distance between the first data point and the data which gives

the highest distance. By normalization of the distance on y as the average distance between the two successive data points, the following equation is obtained:

$$FD = \frac{\log_{10}(L/y)}{\log_{10}(d/y)} \quad (10)$$

In this equation we consider $n=L/y$ as the step number in the curve; therefore, equation (10) can be written as follows:

$$FD = \frac{\log_{10}(n)}{\log_{10}(d/L) + \log_{10}(n)} \quad (11)$$

This equation is known as the Katz method calculating FD of a signal.

4. Proposed Method

The original signal is initially decomposed into different frequency bands using wavelet transform. For multi-component signals which contain different frequency, it is much better we use sub-bands signals which have less frequency variations compared to the original signal. In fact, this pre-processing step can improve the accuracy of proposed method in segmenting multi-component signals.

Then, fractal dimension of the decomposed signals is calculated. For application of adaptive segmentation, we use FD variations in segment boundaries. Therefore, the corresponding G function is as follows:

$$G_m = |FD_{m+1} - FD_m| \quad m = 1, 2, \dots, M-1 \quad (12)$$

where M is total number of the analyzed windows. G function can be normalized as $G_{m/\max(G)}$ to have values between 0 to 1.

Finally, to localize the segment boundaries in a signal, the corresponding G function is thresholded. The local maxima above the threshold value are considered as segments boundaries. One choice is the average value of the G function to assign to the threshold.

5. Performance Assessment

5.1. Synthetic signals

In order to do performance assessment of adaptive segmentation methods, a synthetic multi-component signal has been used (see Figure 1-a). This signal contains seven segments, each of them with different amplitude and frequency.

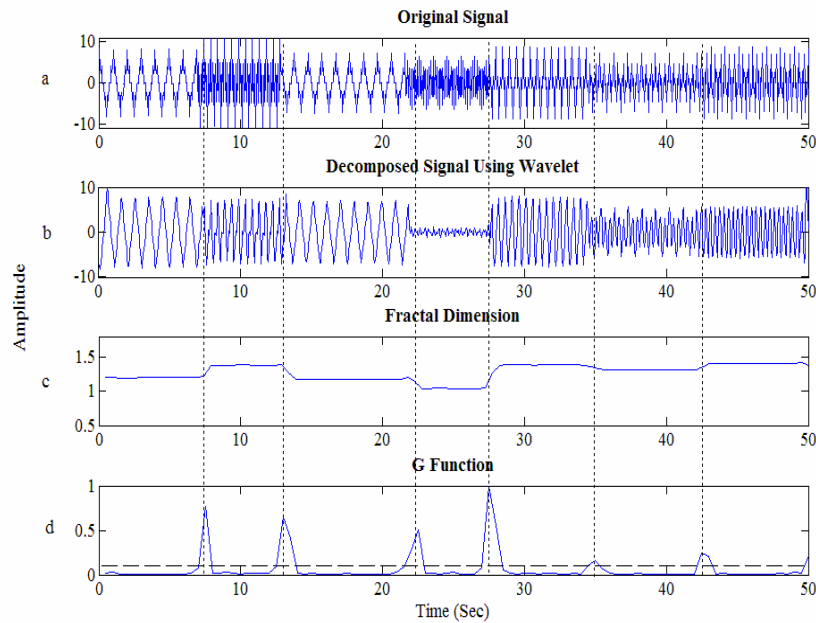


Figure 1. Using the proposed method for signal segmentation. (a) Original signal, (b) Approximate signal after applying one level DWT, (c) Fractal Dimension signal, window length is 1 second and overlapping is 50%, (d) Output of the G function, threshold is 0.15. As can be seen, the boundaries of segments can be truly detected.

We have decomposed the signal using a one level DWT, and the approximate sub-band is depicted in Figure 1-b. In this paper DWT is performed with Daubechies wavelet of order 8. In the next step, fractal dimension of the decomposed signal has been calculated using the Katz algorithm (Figure 1-c). As can be seen from this Figure, fractal dimension in each segment may differ from the fractal dimension of the segment in neighbors; hence, this dimension variation leads detections of segments boundaries. G function is shown in Figure 1-d. The local maxima of G function indicate segments boundaries; hence, as the figure shows all of the 7 segments can be easily identified after thresholding (segment boundaries are demonstrated using vertical dash lines).

Two parameters should be considered in calculating the fractal dimension of a signal [13]. These parameters are: window length, percentage of overlapping for the successive windows. A large window length may results in missing some segments boundaries. Small length of window causes more calculation load. High overlapping increases the computational time, and may cause to have more false segment detection[11].

The signal in Figure 1-a is also segmented using the existing method [9] and the result is shown in Figure 2. The original signal has been decomposed using a one level DWT and the result is shown in Figure 2-b. Figure 2-c represents the output of NEO. Figure 2-d is the result of applying sliding window on the output of NEO. It can be seen that NEO operator, due to cross-term problem, creates false segment boundaries (False Segment boundaries are depicted by arrows in Figure 2-d).

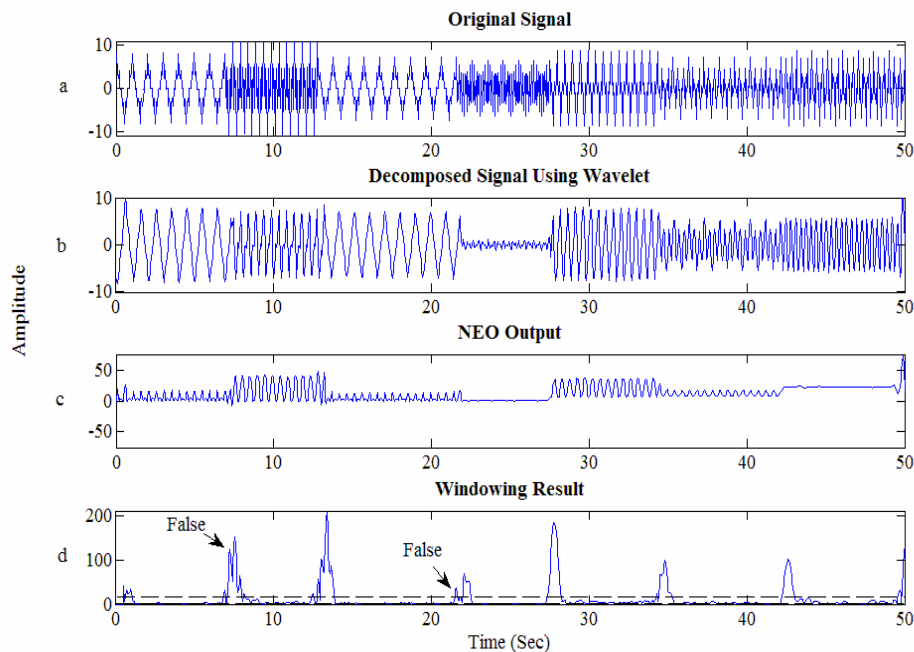


Figure 2. Segmenting the signal in Figure 2-a using a exiting approach: (a) Original Signal, (b) Approximate signal after applying one level DWT, (c) output of NEO, (d) results of applying the sliding window on the output of the NEO, window length is 1 second. As can be seen, this method creates some false segment boundaries.

5.2. Real EEG

To further evaluate the performance of the proposed adaptive segmentation method, it was applied on real EEG data shown in Figure 3-a. we have decomposed the signal using a five level DWT, and the approximate sub-band is shown in Figure 3-b. FD of the decomposed signal and G function are computed and the results are shown in Figures 3-c and Figure 3-d respectively. After thresholding, the local maxima in G function (see Figure 3-d) clearly represent segment boundaries.

The real EEG signal in Figure 3-a is also segmented using the existing method [9] and the result is shown in Figure 4. The approximate signal for the signal in Figure 4-a is shown in Figure 4-b. Figure 4-c shows the output of NEO, and Figure 4-d is the result of applying sliding window on the output of NEO. As can be seen form Figure 4-d, this method creates false boundaries compared to proposed method (false boundaries are shown by arrows in Figure 4-d).

6. Conclusion

In this paper, a new method is introduced for adaptive segmentation of non-stationary signals using wavelet transform and fractal dimension. The proposed method was applied on both the synthetic signal and real EEG data, and the results indicate that the proposed method has a higher accuracy for segmenting non-stationary signals compared to existing method.

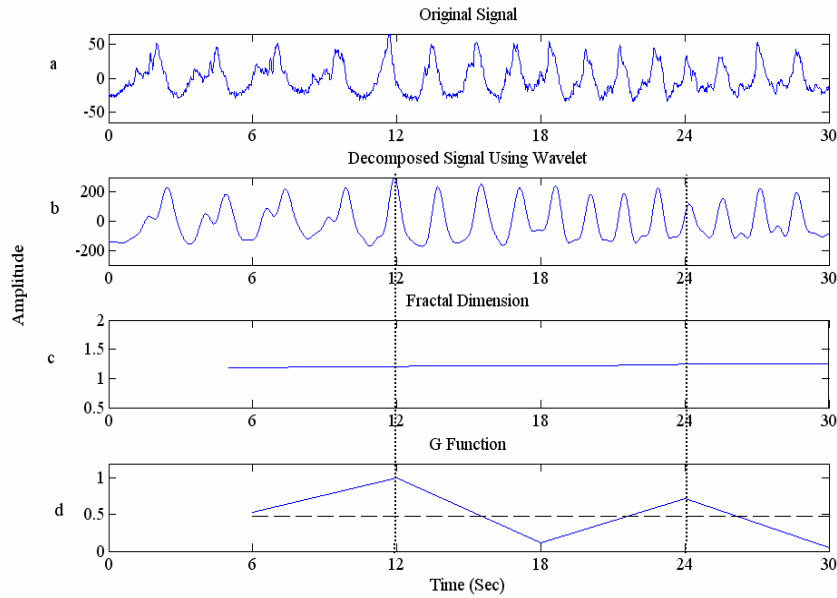


Figure 3. Using the proposed method for signal segmentation. (a) Original signal, (b) Approximate signal after applying five levels DWT, (c) Fractal Dimension signal, window length is 3 second and overlapping is 50%, (d) Output of the G function, threshold is 0.48. As can be seen, the boundaries of segments can be truly detected.

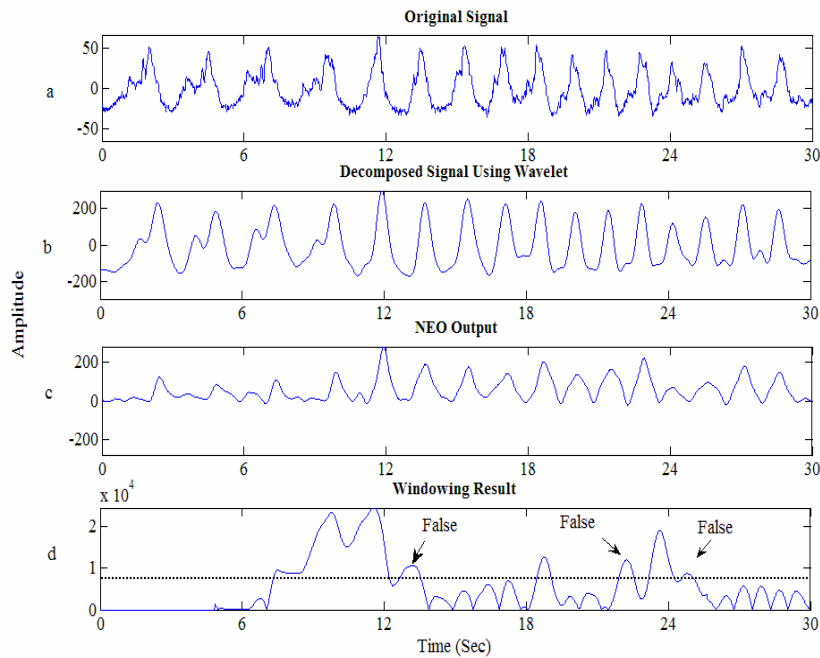


Figure 4. Segmenting the signal in Figure 4-a using a exiting approach: (a) Original Signal, (b) Approximate signal after applying five levels DWT, (c) output of NEO, (d) results of applying the sliding window on the output of the NEO, window length is 3 second. As can be seen, this method creates some false segment boundaries.

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